A Heuristic Algorithm for the Truckload and Less-Than-Truckload Problem

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Abstract

The delivery of goods from a warehouse to local customers is an important and practical problem of a logistics manager. In reality, we are facing the fluctuation of demand. When the total demand is greater than the whole capacity of owned trucks, the logistics managers may consider using an outsider carrier.

Logistics managers can make a selection between a truckload (a private truck) and a less-than-truckload carrier (an outsider carrier). Selecting the right mode to transport a shipment may bring significant cost savings to the company.

In this paper, we address the problem of routing a fixed number of trucks with limited capacity from a central warehouse to customers with known demand. The objective of this paper is developing a heuristic algorithm to route the private trucks and to make a selection of less-than-truckload carriers by minimizing a total cost function. Both the mathematical model and the heuristic algorithm are developed. Finally, some computational results and suggestions for future research are presented.

***Keywords*:** Vehicle routing; Heuristics; 0-1 integer programming; Logistics

1. Introduction

The delivery of goods from a warehouse to local customers is an important and practical problem of a logistics manager. In many sectors of the economy, transportation costs amount for a fifth or even a quarter (lumber, wood, petroleum, stone, clay, and glass products) of the average sales dollars [1].

Logistics managers can make a selection between a truckload (a private truck) and a less-than-truckload carrier (an outsider carrier). A private truck allows a company to consolidate several shipments, going to different destinations, in a single truck. A less-than-truckload carrier usually assumes the responsibility for routing each shipment from origin to destination. The freight charged by a less-than-truckload carrier is usually much higher than the cost of a private truck. Selecting the right mode to transport a shipment may yield significant cost savings to the company.

Our motivation for this study stems from observations on a local logistics company. This company owns different types of trucks and main business of this company is delivering foods and beverages to wholesalers. Since the business hours of the wholesalers are fixed, the delivery time window constraint is not major concern for this company. But, this company is facing the fluctuation of demand within a year. When the demands are greater than the whole capacity of owned trucks during the peak season, there are two ways to deal with this situation. One is asking truck drivers to work overtime; the other is using the outsider carriers. Since the overtime cost is much higher than that of using an outsider carrier, the logistics managers may consider using an outsider carrier.

In this paper, we address the problem of routing a fixed number of trucks with limited capacity from a central warehouse to customers with known demand. The objective of this paper is developing a heuristic algorithm to route the private trucks and to make a selection of less-than-truckload carriers by minimizing a total cost function.

The literature on vehicle routing problem has been concerned almost exclusively with heuristics. Several families of heuristics have been proposed for the VRP. These can be broadly classified into two main classes: classical heuristics developed mostly between 1960 and 1990, and meta-heuristics whose growth has occurred in the last decade [2]. In general, the classical heuristics are of four types: (1) tour building heuristics, (ii) tour improvement heuristics, (iii) two-phase method, (iv) incomplete optimization methods.

The most often mentioned tour building heuristics is the Clarke and Wright method [3]. There have been many modifications to the basic Clarke and Wright method. Gaskell [4] and Yellow [5] independently introduced the concept of a modified savings given by S_{ij}-θC_{ij} where is θ a scalar parameter. One can change emphasis on the cost of travel between two nodes by varying θ.

The tour improvement heuristics are based on the Lin [6] and Lin-Kernighan [7] heuristics for the traveling salesman problem. Christofides and Eilon [8] have modified this heuristic for vehicle routing problem. Two phase methods include those of Gillett and Miller [9], and Christofides et al. [10]. The example of a heuristic based on incomplete optimization is the tree-search method reported in [10].

The metaheuristics, presented below, is restricted to tabu search methods since these have been proved the most successful metaheuristc approach. Over the past decade, tabu search have been applied to the VRP by several authors. Osman [11], Taillard [12], Gendreau et al. [13], Rochat and Taillard [14], Xu and Kelly [15], and Rego and

Roucairol [16] all obtained quite satisfactory results.

Very little research has examined the problem of selecting between a less-than-truckload and truckload carrier. Ball et al. [17] consider a fleet planning problem for long-haul deliveries with fixed delivery locations and an option to use an outside carrier. Agarwal [18] considers the static problem with a fixed fleet size and an option to use an outside carrier. Klincewicz et al. [19] develop a methodology to address the fleet size planning and to route limited trucks from a central warehouse to customers with random daily demands.

In general, our research described here differs from previous fleet planning or vehicle routing in that it modifies the Clarke and Wright method by shifting from distance to cost and also incorporates the fixed cost of different types of trucks into the model; it allows the permutations of the three improvement procedures that will result in best results; it simultaneously considers the determination of routing a heterogeneous fleet vehicles and the selection of less-than-truckload carriers; it also presents a mathematical model for solving the problem.

This paper is organized as follows. Next section formulates the mathematical model for our problem. Section 3 presents the heuristic algorithm. Some computational results are reported in Section 4. Finally some concluding remarks and suggestions for future research are provided in Section 5.

2. Mathematical model

To simplify the analysis, we formulate our mathematical model based on the following assumptions:

1. We consider one warehouse system; all trucks start at the warehouse and return back

to the warehouse.

2. The requirements of all the customers are known; the requirement of each customer cannot exceed the truck capacity;
3. Each customer is served by one truck (either by the private truck or the less-than-truckload carrier); the requirements of all the customers must be met.
4. We restrict ourselves to delivery only.
5. The cost of operating the truck fleet consists of fixed cost and variable cost. Principal cost items in fixed cost include personnel, insurance, and truck depreciation. The main item of variable cost is fuel. It is usually proportional to the distance of truck traveled.

In the following we present an integer programming model and relevant notations：

$$\min \quad z = \sum_{k}^{m} FC_k + \sum_{i}^{n}\sum_{j}^{n}\sum_{k}^{m} C_{ijk} X_{ijk} + \sum_{i}^{n} CL_i L_i$$

subject to

$$\sum_{k}^{m} Y_{0k} = m \qquad (k = 1,\cdots,m) \tag{1}$$

$$\sum_{k}^{m} Y_{ik} + L_i = 1 \qquad (i = 1,\cdots,n) \tag{2}$$

$$\sum_{i}^{n} q_i Y_{ik} \le Q_k \qquad (i = 1,\cdots,n\ ;\ k = 1,\cdots,m) \tag{3}$$

$$\sum_{j}^{n} X_{ijk} = Y_{ik} \qquad (i = 1,\cdots,n\ ;\ k = 1,\cdots,m) \tag{4}$$

$$\sum_{j}^{n} X_{jik} = Y_{ik} \qquad (i = 1,\cdots,n\ ;\ k = 1,\cdots,m) \tag{5}$$

$$\sum_{i}\sum_{j\in s} X_{ijk} \le |S|-1 \quad for \quad all\, S \subseteq \{2,\cdots,n\} \ (k = 1,\cdots,m) \tag{6}$$

$$X_{ijk} \in \{0,1\};\ Y_{ik} \in \{0,1\};\ L_i \in \{0,1\}$$

$$(i = 0,\cdots,n\ ;\ j = 0,\cdots,n\ ;\ k = 1,\cdots,m)$$

i: { i = 0,…,n }, the index set of customers (let the index 0 denote the warehouse);

j: { j = 0,…,n }, the index set of customers;

k: {k = 1,…,m }, the index set of trucks;

n: the number of customers;

m: the number of trucks;

$$X_{ijk} = \begin{cases} 1 & \text{if truck k travels from customer i to customer j} \\ 0 & \text{otherwise} \end{cases}$$

$$L_i = \begin{cases} 1 & \text{if the demand of customer i is satisfied by the less-than-truckload carrier} \\ 0 & \text{otherwise} \end{cases}$$

$$Y_{ik} = \begin{cases} 1 & \text{if the demand of customer i is satisfied by the private vehicle k} \\ 0 & \text{otherwise} \end{cases}$$

FC_k: fixed cost of private truck k

C_{ijk}: the cost of truck k traveling from customer i to customer j

CL_i: the cost charged by the less-than-truckload carrier for serving customer i

q_i: the demand of customer i

Q_i: the capacity of private truck i

The objective is to route the private trucks and to make a selection of less-than-truckload carriers by minimizing a total cost function.

Constraints (1) ensure that all trucks have been assigned to customers.

Constraints (2) ensure that each customer is served either by the private truck or the less-than-truckload carrier.

Constraints (3) are the truck capacity constraints.

Constraints (4) and Constraints (5) ensure that a truck arrives at a customer and also leaves that location.

Constraints (6) serve as subtour-breaking constraints.

3. Heuristic algorithm

In this section we describe an algorithm, called TL-LTL, for solving the vehicle routing and the selection of less-than-truckload carriers problem. The heuristic algorithm can be decomposed into three main steps. In the following we describe algorithm TL-LTL by examining its main steps separately.

3.1 Selection step

The first step of algorithm TL-LTL requires the selection of a group of customers, who will be served by the less-than–truckload carriers. In this step, we will check if the total demand is greater than the whole capacity of owned trucks**.** If the answer is not, we will skip this step and implement next step directly.

In order to minimize the total cost, we have to design a procedure that can achieve this goal. In reality, the freight charged by the less-than-truckload carrier is usually higher than the cost handled by a private truck. It is obvious that we should order the customers in ascending order based on the freight charged by the less-than-truckload carrier and choose the customers with the lowest cost.

The detail for selecting the customers is described as follows:

(1) Calculate the total demand for all customers.

(2) Calculate the whole capacity of owned trucks.

(3) If the total demand for all customers is greater than the whole capacity of owned trucks, go to step (4) otherwise skip this procedure.

(4) Subtract the whole capacity of own trucks from the total demand for all customers, which is the unsatisfied truck capacity.

(5) Order the customers in ascending order based on the freight charged by the less-than-truckload carrier. Starting at top of the list, do the following.

(6) Sum up the demand of each customer until the total demand is greater than the

unsatisfied truck capacity. The corresponding customers will be served by the-less-than-truckload carrier; the remaining customers in the list will be served by private trucks and will be used for constructing initial solution.

3.2 Initial solution construction

The Clarke and Wright's savings algorithm is used to solve this problem by making two modifications. The first modification to the algorithm is a shift in criterion from distance to cost. The second modification of the Clarke and Wright formulation is a change in the savings calculation.

The mathematical relationship of the savings of linking two customers is a function of the mix of a less-than-truckload carrier and a private truck that serve customers. There are three possible mixes serving a pair of customers:(1) two less-than-truckload carriers; (2) a private truck and a less-than-truckload carrier; (3) two private trucks.

Before explaining the revised savings calculation, we list the relevant notations as follows:

S_{ij} = savings from consolidating shipments to customer i and j into the same truck.

LTL_i = the total cost charged by the less-than-truckload carrier for serving customer i.

TL_{ij} = the total cost of a private truck that travels from warehouse to customer i, then from customer i to customer j and finally returns back to warehouse.

FC(Z) = the fixed cost of the smallest truck that can serve a demand of Z

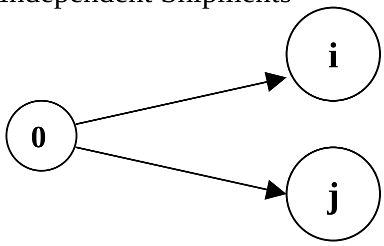
d_{ij} = the distance from customer i to customer j.

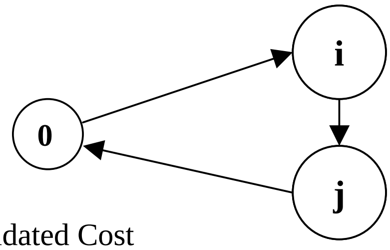
v = the cost of traveling a mile for private truck($/per mile).

Figure 1 illustrates the revised savings calculation from linking two customers under each of the three possible mixes.

1.Carrier mix serving customer i and j: Less-than-truckload and Less-than-truckload

Independent Shipments | Consolidated Shipments





Independent Cost: | Consolidated Cost

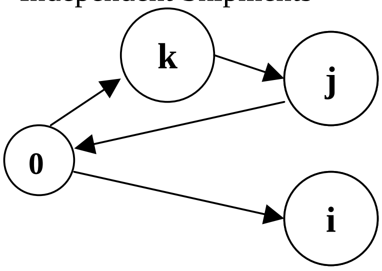
$LTL_i + LTL_j$ | TL_{ij}

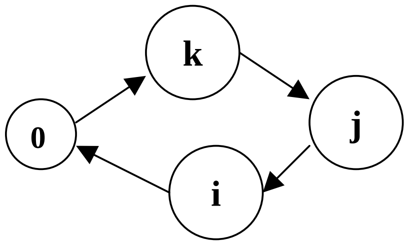
Revised Savings $S_{ij} = LTL_i + LTL_j - TL_{ij}$

$= LTL_i + LTL_j - FC(Z_i + Z_j) - (d_{0i} + d_{ij} + d_{j0})\ v$

2.Carrier mix serving customer i and j:Truckload and Less-than-truckload

Independent Shipments | Consolidated Shipments





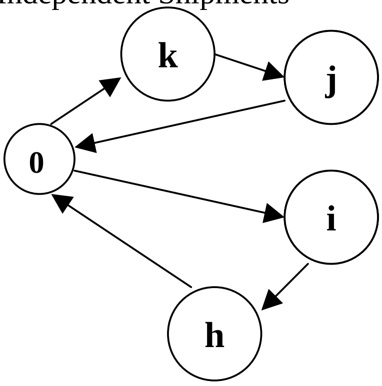
Independent Cost: | Consolidated Cost

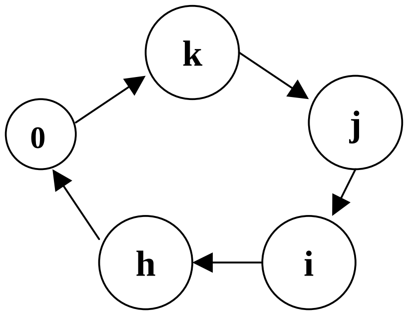
$FC(Z_k + Z_j) + (d_{0k} + d_{kj} + d_{j0})\ v + LTL_i$ | $FC(Z_k + Z_j + Z_i) + (d_{0k} + d_{kj} + d_{ji} + d_{i\,0})\ v$

Revised Savings $S_{ij} = LTL_i + FC(Z_k + Z_j) - FC(Z_k + Z_j + Z_i) + (d_{j0} - d_{ji} - d_{i\,0})\ v$

3.Carrier mix serving customer i and j: Truckload and Truckload

Independent Shipments | Consolidated Shipments





Independent Cost: | Consolidated Cost

$FC(Z_k + Z_j) + (d_{0k} + d_{kj} + d_{j0})\ v$ | $FC(Z_k + Z_j + Z_i + Z_h) + (d_{0k} + d_{kj} + d_{ji} + d_{ih} + d_{h0})\ v$

$+FC(Z_i + Z_h) + (d_{0i} + d_{ih} + d_{h0})\ v$

Revised Savings $S_{ij} = FC(Z_k + Z_j) + FC(Z_i + Z_h) - FC(Z_k + Z_j + Z_i + Z_h) + (d_{j0} + d_{0i} - d_{ji})\ v$

Figure 1. Savings calculation from consolidating two customers.

The detail for constructing the initial solution is described as follows:

(1) Calculate the savings for all pairs customers based on revised savings scenario 1 in Figure 1.

(2) Order the savings in descending order. Starting at top of the list, do the following.

(3) Find the feasible link in the list which can be used to extend one of the two ends of the currently constructed route.

(4) If the route cannot be expanded further, terminate the route. Choose the first feasible link in the list to start a new route.

(5) Repeat Steps (3) and (4) until no more links can be chosen.

(6) Output all the temporary single-customer routes (served by the less-than-truckload carriers) and multi-customer routes.

(7) Calculate the savings for single-customer routes based on revised savings scenario 2 in Figure 1.

(8) Order the savings in descending order. Starting at top of the list, do the following.

(9) Find the feasible link in the current multi-customer routes which can be used to extend the route.

(10) If the route cannot be expanded further, terminate the route.

(11) Repeat Steps (9) and (10) until no more links can be chosen.

(12) Output all the routes.

3.3 Refining procedure

A refining procedure is applied to the solution obtained through the initial solution step. This procedure is composed of a succession of intra-route and inter-route arc exchanges.

3.3.1 Intra-route improvement

Each route is improved by applying a refining procedure which considers all the feasible exchanges of two arcs belong to the route (the so called intra-route two-exchanges, Toth and Vigo [20]). The procedure is similar to those described in Christofides and Eilon [8] and Kindervater and Savelsbergh [21]. Given a route, a two-exchange is obtained by replacing arcs (m, n) and (p, q) with arcs (m, p) and (n, q), as illustrated in Figure 2.

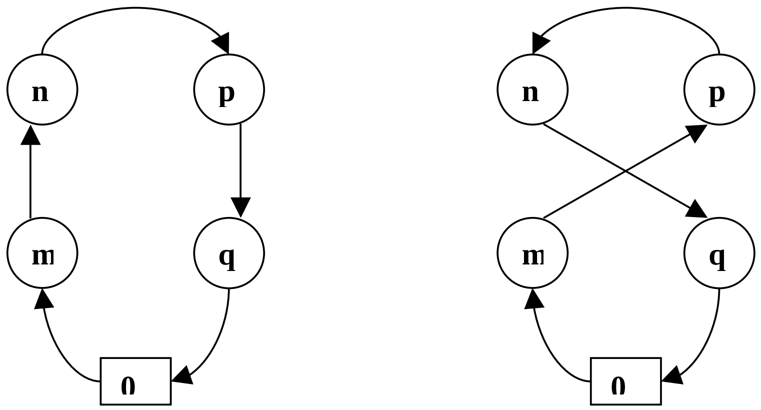


Figure 2. Example of intra-route two-exchanges.

3.3.2 Inter-route improvement

In this step, a set of routes is obtained by using further local search procedures. These procedures are based on the so called inter-route one-exchanges and two exchanges, illustrated in Figure 3 and Figure 4, respectively.

For each node m, belonging to route a, the one-exchange corresponding to its insertion after node p, belonging to route b, is obtained by removing arcs (l, m), (m, n) and (p, q), and replacing them with arcs (l, n), (p, m) and (m, q), as illustrated in Figure 3.

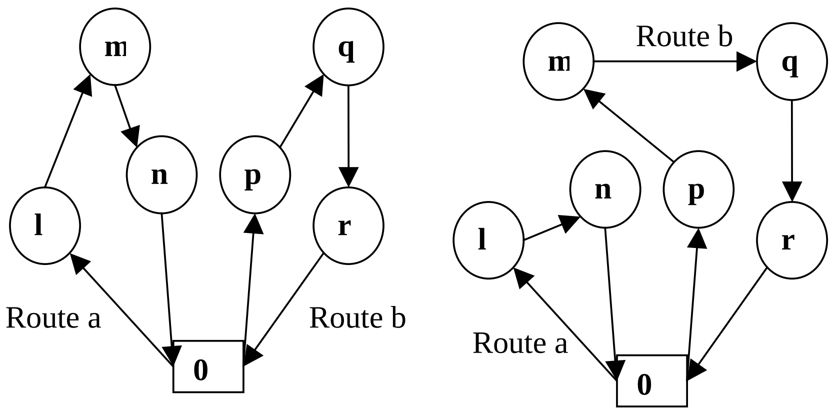


Figure 3. Example of inter-route one-exchange.

For each node m, belonging to route a, the two-exchange corresponding to its exchange with node q, belonging to route b, is obtained by removing arcs (l, m), (m, n), (p, q) and (q, r), and replacing them with arcs (l, q), (q, n), (p, m) and (m, r), as illustrated in Figure 4.

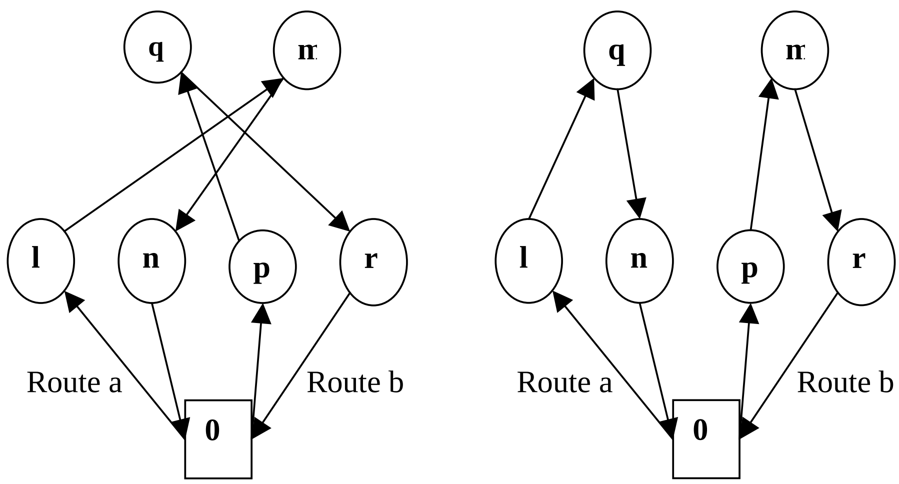


Figure 4. Example of inter-route two-exchanges.

3.3.3 Search Procedure

A search procedure is designed in searching for a better solution. From the results of extensive experiments which are not shown here, we know that the implementation sequence of intra-route and inter-route improvement procedure might have impacts on the quality of solution.

The improvement procedures mentioned above include intra-route two-exchanges, inter-route one-exchanges and two exchanges. The possible permutations of three different improvement procedures are only six, so a loop procedure consisting of arranging the possible sequences of intra-route and inter-route improvement is applied on the solution obtained in the initial solution construction phase. The purpose of this loop procedure is in a sense to similar to the tabu search method to escape from a local minimum. Once a better solution is found after finishing improvement phase, the best solution record is updated. We repeat the above improvement processes until all possible permutations of three different improvement procedures have been implemented.

4. Computational results

In this section, we summarize our computational results on five test problems. The detailed data associated with five examples are given in the Appendix. The solutions produced by the heuristic algorithm are compared with the optimal results from the mathematical model. The heuristic algorithm was written in FORTRAN language and the mathematical model was solved using the software LINDO version 6.1. Both of them were implemented on a PC with a 2000 MHz processor. Computational results on five test problems are reported in Table 1.

Table 1. Results for five test problems

Test problem		Routes	Total Cost	CPU(i) Time	% larger than the best solution
1	Heuristic Algorithm	1-3-5-4-1 1-6-1 Customer 2 is served by LTL	387.5	3.14	0
	Mathematical Model	1-3-5-4-1 1-6-1 Customer 2 is served by LTL	387.5	1.1	
2	Heuristic Algorithm	1-9-8-7-5-11-1 1-2-4-3-10-1 Customer 6 is served by LTL	631	4.58	7.67
	Mathematical Model	1-4-3-10-11-5-1 1-2-9-8-7-1 Customer 6 is served by LTL	586	34.5	
3	Heuristic Algorithm	1-3-2-4-11-10-1 1-12-15-8-13-1 1-16-6-14-9-7-1 Customer 5 is served by LTL	900	5.88	0
	Mathematical Model	1-8-12-4-2-3-1 1-7-13-10-11-1 1-9-15-14-16-6-1 Customer 5 is served by LTL	900	578	
4	Heuristic Algorithm	1-17-16-4-3-2-7-14-10-6-5-9-1 1-22-20-19-21-15-18-23-12-8-13-1 Customer 11 is served by LTL	1681.5	8.42	1.81
	Mathematical Model	1-20-23-21-19-15-18-16-17-4-3-2-8-10-13-1 1-7-14-12-6-5-9-22-1 Customer 11 is served by LTL	1651.5	830	
5	Heuristic Algorithm	1-19-24-9-15-30-28-27-29-1 1-11-12-13-10-18-8-14-17-16-1 1-20-21-23-7-26-25-2-6-5-4-1 Customers 3 and 22 are served by LTL	1917	11.06	0.86
	Mathematical Model	1-3-5-2-1 1-22-18-17-27-29-28-26-25-23-21-1 1-4-6-7-30-16-14-8-10-15-9-13-12-11-1 Customers 19, 20 and 24 are served by LTL	1900.5	2406	

(i) All times are in seconds; the results were obtained on a PC running at 2000 MHz.

For the first and the third test problems, our heuristic algorithm obtains the optimal solution. As shown in Table 1, both the mathematical model and the heuristic algorithm yield the same total cost $387.5 and $900, respectively. The only difference between two approaches in the third test problem is in that each approach arranges customers in different routes and in different sequences.

Computationally, exact algorithm for the VRP is restricted to solving problems of only up to about 25 customers. For five test problems, the solution time of mathematical model increased quickly with problem size. On the other side, our heuristic algorithm required very little time to solve the problem. Every problem took only a few seconds. The CPU time of test problems is not very sensitive to problem size.

In order to test whether the solution time of our algorithm is not sensitive to larger size of problem, we have solved additional three test problems with the customer size of 51, 76 and 101, respectively. Because the VRP is very difficult to solve with mathematical model even for relatively small size instances, only the average computation times to run the heuristic are reported. These results are presented in Table 2. Though the solution's time increased with problem size, it is obvious that the solution's time increase gradually without rapid growth.

Table 2. Results for larger size of test problems

Test problem (i)	CPU(ii) Time
1 [E-n51-K5]	27.84
2 [E-n76-K7]	84.48
3 [E-n101-K8]	192.48

(i) Test problem 1 and 3 can be found in Christofides and Eilon [8]; Test problem 2 can be found in Gillett and Miller [9].

(ii) All times are in seconds; the results were obtained on a PC running at 2000 MHz.

5. Conclusions

The delivery of goods from a warehouse to local customers is an important and practical problem of a logistics manager. In this paper, we develop both the mathematical model and the heuristic algorithm for solving the less-than-truckload and truckload problem. Some computational results are presented. Our heuristic algorithm obtains the optimal or near-optimal solutions in an efficient way in terms of time and accuracy.

As for further research, a wide range of test problems should be performed. It would be interesting to see if other intelligent optimization techniques, such as Tabu Search, Genetic Algorithms, simulated Annealing and Neural Networks, can be modified to solve this problem and even provide better results.

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APPENDIX

Details of problem 1					Details of problem 2					Details of problem 3				
No.	x	y	Q	LTL	No.	x	y	q	LTL	No.	x	y	Q	LTL
1	35	35	0	0	1	30	40	0	0	1	40	40	0	0
2	41	49	10	90	2	37	52	7	78	2	22	22	18	150
3	35	17	7	108	3	49	49	30	126	3	36	26	26	84
4	55	45	13	132	4	52	64	16	192	4	21	45	11	114
5	55	20	19	150	5	20	26	9	102	5	45	35	30	42
6	15	30	26	120	6	40	30	21	84	6	55	20	21	150
					7	21	47	15	66	7	33	34	19	54
					8	17	63	19	156	8	50	50	15	84
					9	31	62	23	132	9	55	45	16	90
					10	52	33	11	138	10	26	59	29	138
					11	51	21	5	168	11	40	66	26	156
										12	55	65	37	174
										13	35	51	16	72
										14	62	35	12	132
										15	62	57	31	162
										16	62	24	8	162

Problem 1

Warehouse co-ordinates(35,35);
Customer demands (q) in cwt.

Vehicle	Capacity	Fixed Cost
1	40 cwt	60
2	30 cwt	50

The variable cost for private Vehicles is $1.5/per mile

Problem 2

Warehouse co-ordinates(30,40);
Customer demands (q) in cwt.

Vehicle	Capacity	Fixed Cost
1	75 cwt	120
2	65 cwt	100

The variable cost for private vehicles is $1.5/per mile

Problem 3

Warehouse co-ordinates(40,40);
Customer demands (q) in cwt.

Vehicle	Capacity	Fixed Cost
1	110 cwt	150
2	100 cwt	140
3	90 cwt	130

The variable cost for private vehicles is $1.5/per mile

Details of problem 4				
No.	x	y	Q	LTL
1	266	235	0	0
2	295	272	125	282
3	301	258	84	246
4	309	260	60	294
5	217	274	500	372
6	218	278	300	384
7	282	267	175	210
8	242	249	350	162
9	230	262	150	270
10	249	268	1100	222
11	256	267	4100	198
12	265	257	225	132
13	267	242	300	42
14	259	265	250	180
15	315	233	500	294
16	329	252	150	390
17	318	252	100	324
18	329	224	250	378
19	267	213	120	132
20	275	192	600	258
21	303	201	500	300
22	208	217	175	360
23	326	181	75	480

Warehouse co-ordinates (266,235);

Customer demands (q) in cwt

Vehicle	Capacity	Fixed Cost
1	4500 cwt	250
2	4000 cwt	200

The variable cost for private Vehicles is $1.5/per mile

Details of problem 5									
No.	x	y	q	LTL	No.	x	y	q	LTL
1	162	354	0	0	21	180	360	300	114
2	218	382	300	372	22	159	331	1500	138
3	218	358	3100	336	23	188	357	100	156
4	201	370	125	252	24	152	349	300	66
5	214	371	100	324	25	215	389	500	378
6	224	370	200	384	26	212	394	800	384
7	210	382	150	330	27	188	393	300	276
8	104	354	150	348	28	207	406	100	408
9	126	338	450	234	29	184	410	150	360
10	119	340	300	270	30	207	392	1000	348
11	129	349	100	198					
12	126	347	950	216					
13	125	346	125	222					
14	116	355	150	276					
15	126	335	150	240					
16	125	355	550	222					
17	119	357	150	258					
18	115	341	100	288					
19	153	351	150	54					
20	175	363	400	90					

Warehouse co-ordinates (162,354);

Customer demands (q) in cwt

Vehicle	Capacity	Fixed Cost
1	4500 cwt	250
2	4000 cwt	200
3	3500 cwt	180

The variable cost for private Vehicles is $1.5/per mile