Staff Scheduling with Two Consecutive Days off during the Weekend and Work Stretch Constraints

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Abstract

Days-off scheduling in many service and manufacturing organizations such as hospitals, police stations, and steel plants working seven days a week is a critical management activity. The purpose of this study is to formulate a mathematical programming model solving staff scheduling problem with two consecutive days off during the weekend and work stretch constraints efficiently.

Keywords: Integer programming; Days-off scheduling

1. Introduction

The objective of days-off scheduling problem is to assign employees to work and non-work days through the week. Days-off scheduling in many service and manufacturing organizations such as hospitals, police stations, paper mills and steel plants working seven days a week has received considerable attention in the past.

The motivation of our study is stemmed from a local police substation. In this paper we focus on the days-off scheduling problem with two consecutive days off during the weekend and work stretch constraints. Several studies in this area have been devoted to other version of this problem. We present the relevant work in the following paragraphs.

Tibrewala, Philippe and Browne [7] investigated the case in which variable demand for staff during the week and all employees must be assigned two consecutive days off in each week. In this study, both provisions for weekends and constraints on work stretches are not considered. Rothestein [9] formulated a similar problem in which nonconsecutive days off are considered and less desirable than consecutive days off. Brownell and Lowrre [2] explored a case where the staff requirements are different between weekdays and weekend and day off constraints in various combinations were examined. No work stretch is included in this study.

Rothe and Wolfe [8] described a particular application in which both work stretches and

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long weekend factors were considered, but no comprehensive model was presented. Burns [3] proposed a solution to the variable demand problem where each employee must be assigned one out of two weekends off. Work stretch constraint is not explored in this study.

Baker, Burns and Carter [1] examined a case with a fixed daily weekday demand, a fixed daily weekend demand and any A out of B weekends off. By exploring special structure in the problem, authors deduced a flexible, modular technique for building optimal schedules. In this paper, the week is defined as Sunday through Saturday; this definition actually provides more flexibilities than a Monday-through-Sunday week in situations where regulations govern weekends off. Burns and Carter [4] generalized the work to incorporate both varying staff requirements and off time requirements, and presented a linear time algorithm generating schedules satisfying all the objectives.

Emmons [5] generalized previous work and presented a generalization of cyclical workforce schedule. The minimum worker need for a seven-days-a-week operation is proposed. Using this workforce, the generalized cyclical scheduling procedure produces a feasible schedule. As previous work, the week is defined as Sunday through Saturday. Both weekends off and work stretch constraints are considered. Emmons and Fuh [6] explored a staff scheduling problem with off-day and off-weekend constraints. In this research, staff requirements are assumed constant Monday through Friday. Furthermore, full time employ can be supplemented by part time workers. Authors presented formulas for the most economical mix of workers and an algorithm that produces a feasible schedule.

Most of past studies are based on heuristic algorithms. In practice, heuristic algorithms are often developed because they tend to be faster in terms of time. However, sometimes heuristic approaches cannot produce a feasible schedule easily if a lot of constraints are involved. Mathematical programming technique can be viewed as an effective approach to these complex situations if it can produce a feasible solution in a reasonable time. This is the philosophy adopted in this paper.

The purpose of this study is to formulate a mathematical programming model for solving the problem described above efficiently. The major differences between our work and past studies are in that a number of realistic scheduling constraints (weekend off, work stretch, variable daily demand and minimum workforce) are considered and a week is defined as Monday through Sunday. No research with identical problem scenario has been conducted.

This rest of paper is organized as follow. The formal presentation of the model is explained in Section 2. Section 3 describes the numerical solution. The final section provides some concluding remarks and suggestions for future research.

2. Formulation

2.1 Problem Scenario

The problem scenarios of our model are listed as follows:

1. There are forty policemen in a local police substation.
2. Each policeman must be given two consecutive days off per week.
3. There are four weeks in the planning period.
4. A week is defined as Monday through Sunday.
5. One third policemen are allowed to be off-duty everyday (i.e., there is a requirement for two thirds policemen per day, seven days a week.)
6. Each policeman must be given at least one weekend off.
7. No policeman shall be assigned a work stretch (i.e., a sequence of consecutive work days) of more than seven days.

2.2 Decision variable and Parameters

$$X_{ijk} = \begin{cases} 1 & \text{if policeman } i \text{ is assigned to shift } k \text{ during the } j\text{th week} \\ 0 & \text{otherwise} \end{cases}$$

i: { i = 1,…, m }, the index set of policemen available for scheduling everyday, where m is the total number of policemen available for scheduling everyday.

j: { j = 1,…, n }, the index set of weeks, where n is the number weeks in the planning period.

k: {k = 1,…, p }, the index set of work shifts, where p is the number of work shifts in the planning period. The detail of work shifts are described as follows.

Table 1 work shift table

shift	Workdays	off days
1	Mon Tue Wed Thr Fri	Sat Sun
2	Wed Thr Fri Sat Sun	Mon Tue
3	Thr Fri Sat Sun Mon	Tue Wed
4	Fri Sat Sun Mon Tue	Wed Thr
5	Sat Sun Mon Tue Wed	Thr Fri
6	Sun Mon Tue Wed Thr	Fri Sat

2.3 The objective function

$$\text{Max} \quad z = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} X_{ijk}$$

The objective function maximizes the total available workforce during the planning period.

2.4 Constraints

$$\sum_{k=1}^{p} X_{ijk} = 1 \qquad (i = 1, \cdots, m;\ j = 1, \cdots, n) \qquad (1)$$

$$\sum_{j=1}^{n} X_{ij1} \geq 1 \qquad (i = 1, \cdots, m) \qquad (2)$$

$$\sum_{i=1}^{m} X_{ij1} + \sum_{i=1}^{m} \sum_{k=3}^{6} X_{ijk} \geq \frac{2}{3} m \qquad (j = 1, \cdots, n) \qquad (3)$$

$$\sum_{i=1}^{m} X_{ij1} + \sum_{i=1}^{m} \sum_{k=4}^{6} X_{ijk} \geq \frac{2}{3} m \qquad (j = 1, \cdots, n) \qquad (4)$$

$$\sum_{i=1}^{m} \sum_{k=1}^{2} X_{ijk} + \sum_{i=1}^{m} \sum_{k=5}^{6} X_{ijk} \geq \frac{2}{3} m \qquad (j = 1, \cdots, n) \qquad (5)$$

$$\sum_{i=1}^{m} \sum_{k=1}^{3} X_{ijk} + \sum_{i=1}^{m} X_{ij6} \geq \frac{2}{3} m \qquad (j = 1, \cdots, n) \qquad (6)$$

$$\sum_{i=1}^{m} \sum_{k=1}^{4} X_{ijk} \geq \frac{2}{3} m \qquad (j = 1, \cdots, n) \qquad (7)$$

$$\sum_{i=1}^{m} \sum_{k=2}^{5} X_{ijk} \geq \frac{2}{3} m \qquad (j = 1, \cdots, n) \qquad (8)$$

$$\sum_{i=1}^{m} \sum_{k=2}^{6} X_{ijk} \geq \frac{2}{3} m \qquad (j = 1, \cdots, n) \qquad (9)$$

$$X_{ij2} + X_{i\,j+1\,1} \leq 1 \quad (i = 1, \cdots, m;\ j = 1, \cdots, 3) \qquad (10)$$

$$X_{ij2} + X_{i\,j+1\,5} \leq 1 \quad (i = 1, \cdots, m;\ j = 1, \cdots, 3) \qquad (11)$$

$$X_{ij2} + X_{i\,j+1\,6} \leq 1 \quad (i = 1, \cdots, m;\ j = 1, \cdots, 3) \qquad (12)$$

$$X_{ij3} + X_{i\,j+1\,1} \leq 1 \quad (i = 1, \cdots, m;\ j = 1, \cdots, 3) \qquad (13)$$

$$X_{ij3} + X_{i\,j+1\,6} \leq 1 \quad (i = 1, \cdots, m;\ j = 1, \cdots, 3) \qquad (14)$$

$$X_{ij4} + X_{i\,j+1\,1} \leq 1 \quad (i = 1, \cdots, m;\ j = 1, \cdots, 3) \qquad (15)$$

$$\forall X_{ijk} \in \{0,1\}$$

Constraint (1) requires that every policeman will be assigned only one work shift in every week.

Constraint (2) ensures that each policeman must be given at least one weekend off during the planning period.

Constraints (3)-(9) guarantee that the minimum workforce requirement is satisfied from Monday to Sunday, respectively. The formulation of each inequality can be seen easily from the following table.

Table 2 work shift and working days table

Work Shift	Mon	Tue	Wed	Thr	Fri	Sat	Sun
1	▲	▲	▲	▲	▲		
2			▲	▲	▲	▲	▲
3	▲			▲	▲	▲	▲
4	▲	▲			▲	▲	▲
5	▲	▲	▲			▲	▲
6	▲	▲	▲	▲			▲

▲ stands for a working day

For example, constraint (3) consists of two terms. First term includes all policemen assigned to work shift 1. Second term contains all policemen allocated to work shift 3, 4, 5 and 6. It is clear that constraint (3) requiring the workforce on Monday should be greater than or equal to two thirds of total number of policemen available.

With similar reasoning, constraints (4)-(9) impose a similar restriction on minimum workforce requirement from Tuesday to Sunday, respectively.

Inequalities (10)-(15) are work stretch constraints. Those Inequalities require that no policeman shall be assigned a work stretch of more than seven days. Table 3 enumerates all possible successive work shift combinations which will result in a work stretch of more than seven days. Since all decision variables in our model are 0-1 integer, the right hand side of inequalities (10)-(15) is less than or equal to one. It will eliminate all possible cases with a work stretch of more than seven days.

Table 3 possible work shift combinations with a work stretch of more than seven days

Work Shift	M	T	W	T	F	S	S	M	T	W	T	F	S	S	Work stretch
(2, 1)			▲	▲	▲	▲	▲	▲	▲	▲	▲	▲			10 days
(2, 5)			▲	▲	▲	▲	▲	▲	▲	▲			▲	▲	8 days
(2, 6)			▲	▲	▲	▲	▲	▲	▲	▲	▲			▲	9 days
(3, 1)				▲	▲	▲	▲	▲	▲	▲	▲	▲			9 days
(3, 6)				▲	▲	▲	▲	▲	▲	▲	▲			▲	8 days
(4, 1)	▲	▲			▲	▲	▲	▲	▲	▲	▲	▲			8 days

▲ stands for a working day

3. Numerical solution

We demonstrate our model based on the data taken from a local police substation. There are forty policemen in the police substation. Each policeman works exactly five days and has two consecutive days off in every week. According the regulation, the maximum number of consecutive days worked for every policeman is seven days and one third policemen are allowed to be off-duty everyday. Furthermore, in order to increase the morale of every policeman, each policeman should be given at least one weekend off.

This sample problem is solved on a PC. First, we write a program coded in FORTRAN language to produce the objective function and constraints described in our model. There are 960 integer decision variables and 947 constraints included in our problem. Then, we execute the FORTRAN program and put the output into the optimization software LINDO. It takes only 11 seconds to solve this problem on a PC with the speed of 800 MHz. Based on the solution from LINDO, it takes about 5 to 7 minutes to establish the timetable manually. The detail information of the timetable is summarized in Table 4.

From table 4, we can observe that each policeman is assigned exactly one weekend off (the grey area). The work stretch is between one day and seven days, and the available daily work force is equal to or greater than two thirds of total number of policemen available. Next, let us compare our results with the scheduling now being used. The current schedule now used is summarized in Table 5. After discussing with the police officer in charge of scheduling the work-days timetable, we know that the work-days timetable is established based on his experience.

If we examine the Table 5 carefully, there are twelve policemen can be assigned at least one weekend off. Two and six policemen are granted three and two weekends off, respectively; four policemen are assigned one weekend off. The remaining of policemen are not even assigned one weekend off. It is obvious that the schedule from current practice cannot satisfy the objective that each policeman should be assigned at least one weekend off.

The work stretch based on current practice is between one day and five days, and the available daily work force is equal to 28 policemen available. The main advantage of current scheduling is that the work stretch is at most five days which is less than that of our model.

Table 4 A work-days timetable based on our model

	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S
1	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲			▲			▲	▲	▲	▲			▲	▲	▲	▲	▲
2	▲	▲	▲	▲	▲			▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲			▲	▲
3	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲			▲	▲	▲			▲	▲	▲	▲			▲	▲	▲
4			▲	▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲		
5	▲	▲	▲	▲	▲					▲	▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲			▲	▲
6	▲	▲	▲	▲	▲					▲	▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲			▲	▲
7	▲	▲	▲	▲	▲			▲	▲	▲			▲	▲	▲			▲	▲	▲	▲	▲	▲	▲			▲	▲
8	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲					▲	▲	▲	▲	▲			▲	▲	▲	▲	▲
9	▲			▲	▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲		
10	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲			▲			▲	▲	▲	▲			▲	▲	▲	▲	▲
11	▲	▲			▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲			▲	▲	▲			▲	▲
12	▲			▲	▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲		
13	▲	▲			▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲		
14	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲					▲	▲	▲	▲	▲	▲			▲	▲	▲	▲
15	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲		
16	▲	▲			▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲					▲	▲	▲	▲	▲
17	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲					▲	▲	▲	▲	▲	▲	▲			▲	▲	▲
18	▲	▲	▲	▲	▲			▲	▲			▲	▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲
19	▲	▲	▲			▲	▲	▲			▲	▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲		
20	▲	▲	▲	▲	▲			▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲
21	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲			▲	▲			▲	▲	▲	▲	▲			▲	▲	▲
22	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲			▲	▲			▲	▲	▲	▲	▲	▲	▲			▲
23	▲			▲	▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲			▲	▲	▲	▲			▲
24	▲	▲	▲	▲	▲			▲			▲	▲	▲	▲	▲	▲	▲			▲	▲			▲	▲	▲	▲	▲
25	▲			▲	▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲		
26	▲			▲	▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲			▲	▲	▲			▲	▲
27			▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲		
28	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲			▲	▲			▲	▲	▲			▲	▲	▲	▲	▲
29	▲	▲	▲	▲	▲			▲	▲			▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲
30	▲			▲	▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲			▲			▲	▲	▲	▲
31	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲					▲	▲	▲	▲	▲	▲			▲	▲	▲	▲
32	▲	▲	▲	▲	▲			▲	▲	▲	▲			▲			▲	▲	▲	▲	▲	▲	▲			▲	▲	▲
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34	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲		
35	▲	▲			▲	▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲			▲	▲	▲			▲	▲
36	▲			▲	▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲			▲	▲	▲			▲	▲
37	▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲			▲			▲	▲	▲	▲	▲
38	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲		
39	▲	▲	▲	▲	▲					▲	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲			▲	▲
40	▲	▲	▲			▲	▲	▲	▲	▲	▲	▲					▲	▲	▲	▲	▲	▲	▲			▲	▲	▲
total	38	28	26	26	28	26	28	36	28	29	26	26	26	29	34	27	27	26	26	27	33	32	28	31	26	27	26	30

Table 5 A work-days timetable based on current practice

	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S
1	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲
2	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲
3	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲
4			▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	
5		▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲		
6	▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲
7	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲
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16	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲
17			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲	
18		▲	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲		
19	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲			▲
20	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲
21	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲
22	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲
23	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲
24			▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲	
25		▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲		
26	▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲
27	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲
28	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲
29	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲
30	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	
31			▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲		
32		▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲			▲
33	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲
34	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲
35	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲
36	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲
37			▲	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲	
38		▲	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲		
39	▲	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲			▲
40	▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲	▲	▲			▲	▲	▲	▲	▲			▲	▲
total	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28

4. Conclusions

We have proposed a mathematical model for solving staff (policeman) scheduling problem with two consecutive days off during the weekend and work stretch constraints. The major differences between our work and past studies are in that two consecutive days off during the weekend constraint is considered and a week is defined as Monday through Sunday instead of a Sunday-through-Saturday week in our model. No research with identical problem scenario has been conducted. In our study, a sample problem from a local police substation is solved. It has proven that our model can produce an optimal solution efficiently.

The advantage of our mathematical model is that the formulation accommodates a number of realistic scheduling constraints (weekend off, work stretch and minimum workforce) and still yields the optimal solution efficiently. Other advantages of our model are the flexibility and the scale of application. The model is quite flexible. For example, when the minimum workforce requirement and the frequency of weekends off have changed, we can solve the problem simply by modifying the coefficients on the right hand side of inequalities. The extended version of LINDO would solve model with 20,000 0-1 integer decision variables and with this software the model could be expanded to about 714 people.

The model develop in this paper can be easily adapted to other staff scheduling problems, as long as the work schedule for a worker can be characterized by a series of work stretches and the minimum workforce requirement can be specified. As the problem size becomes large, it would be convenient to design a program that can integrate the solution from LINDO and produce the optimal schedule automatically in the future. It would be also interesting to see if a heuristic algorithm can be developed to solve the problem efficiently mentioned in this paper.

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