of one decision cannot be assessed until we know how others are resolved. Such circumstances often lead to quadratic assignment models.

> **11.16** Quadratic assignment models minimize or maximize a quadratic objective function of the form
>
> $$\sum_{i}\sum_{j}\sum_{k \neq i}\sum_{\ell \neq j} c_{i,j,k,\ell}\, x_{i,j}\, x_{k,\ell}$$
>
> subject to assignment constraints **11.14**, where $c_{i,j,k,\ell}$ is the cost (or benefit) of assigning i to j and k to ℓ.

Notice that each objective function term

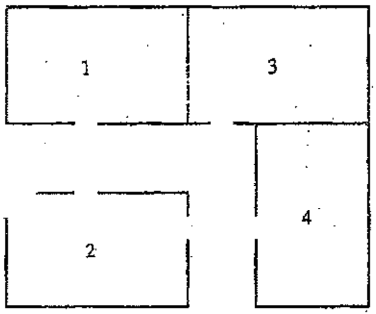
$$c_{i,j,k,\ell} \cdot x_{i,j} \cdot x_{k,\ell}$$

involves two assignment decisions. Cost $c_{i,j,k,\ell}$ is realized only if both $x_{i,j} = 1$ and $x_{k,\ell} = 1$. That is, $c_{i,j,k,\ell}$ applies only if i is assigned to j and k is assigned to ℓ.

Example 11.5: Mall Layout Quadratic Assignment

Some of the most common cases producing quadratic assignment models arise in **facility layout.** We are given a collection of machines, offices, departments, stores, and so on, to arrange within a facility, and a set of locations within which they must fit. The problem is to decide which unit to assign to each location.

Figure 11.3 illustrates with 4 possible locations for stores in a shopping mall. Walking distances (in feet) between the shop locations are displayed in the adjacent table. The 4 prospective tenants for the shop locations are listed in Table 11.5. The table also shows the number of customers each week (in thousands) who might wish to visit various pairs of shops. For example, a projected 5 thousand customers per week will visit both 1 (Clothes Are) and 2 (Computers Aye).



Distance (feet)				
$j \backslash \ell$	1	2	3	4
1	—	80	150	170
2	80	—	130	100
3	150	130	—	120
4	150	100	120	—

Figure 11.3 Mall Layout Example Locations

Mall managers want to arrange the stores in the 4 locations to minimize customer inconvenience. One very common measure is **flow-distance**, the product of flow volumes between facilities and the distances between their assigned locations. For example, if shop 1 (Clothes Are) is located in space 1, and shop 4 (Book Bazaar) is located in space 2, their 7 thousand common customers will have to walk the 80 feet between the locations. This adds $7 \cdot 80 = 560$ thousand customer-feet to the flow-distance.

TABLE 11.5 Mall Layout Example Tenants

Store, i	Common Customers with k (000's) 1	2	3	4
1: Clothes Are	—	5	2	7
2: Computers Aye	5	—	3	8
3: Toy Parade	2	3	—	3
4: Book Bazaar	7	8	3	—

Mall Layout Example Model

Notice that the flow-distance for any pair of shops cannot be computed until we know where both are assigned. This is the assignment combinations characteristic that yields quadratic assignment models.

Using the decision variables

$$x_{i,j} \triangleq \begin{cases} 1 & \text{if shop } i \text{ is assigned to location } j \\ 0 & \text{otherwise} \end{cases}$$

the required quadratic assignment model is

$$\begin{aligned}
\min \quad & 5(80x_{1,1}x_{2,2} + 150x_{1,1}x_{2,3} + 170x_{1,1}x_{2,4} && \text{(shops 1 and 2)} \\
& \quad + 80x_{1,2}x_{2,1} + 130x_{1,2}x_{2,3} + 100x_{1,2}x_{2,4} \\
& \quad + 150x_{1,3}x_{2,1} + 130x_{1,3}x_{2,2} + 120x_{1,3}x_{2,4} \\
& \quad + 170x_{1,4}x_{2,1} + 100x_{1,4}x_{2,2} + 120x_{1,4}x_{2,3}) \\
& 2(80x_{1,1}x_{3,2} + 150x_{1,1}x_{3,3} + 170x_{1,1}x_{3,4} && \text{(shops 1 and 3)} \\
& \quad + 80x_{1,2}x_{3,1} + 130x_{1,2}x_{3,3} + 100x_{1,2}x_{3,4} \\
& \quad + 150x_{1,3}x_{3,1} + 130x_{1,3}x_{3,2} + 120x_{1,3}x_{3,4} \\
& \quad + 170x_{1,4}x_{3,1} + 100x_{1,4}x_{3,2} + 120x_{1,4}x_{3,3}) \\
& 7(80x_{1,1}x_{4,2} + 150x_{1,1}x_{4,3} + 170x_{1,1}x_{4,4} && \text{(shops 1 and 4)} \\
& \quad + 80x_{1,2}x_{4,1} + 130x_{1,2}x_{4,3} + 100x_{1,2}x_{4,4} && (11.12) \\
& \quad + 150x_{1,3}x_{4,1} + 130x_{1,3}x_{4,2} + 120x_{1,3}x_{4,4} \\
& \quad + 170x_{1,4}x_{4,1} + 100x_{1,4}x_{4,2} + 120x_{1,4}x_{4,3}) \\
& 3(80x_{2,1}x_{3,2} + 150x_{2,1}x_{3,3} + 170x_{2,1}x_{3,4} && \text{(shops 2 and 3)} \\
& \quad + 80x_{2,2}x_{3,1} + 130x_{2,2}x_{3,3} + 100x_{2,2}x_{3,4} \\
& \quad + 150x_{2,3}x_{3,1} + 130x_{2,3}x_{3,2} + 120x_{2,3}x_{3,4} \\
& \quad + 170x_{2,4}x_{3,1} + 100x_{2,4}x_{3,2} + 120x_{2,4}x_{3,3}) \\
& 8(80x_{2,1}x_{4,2} + 150x_{2,1}x_{4,3} + 170x_{2,1}x_{4,4} && \text{(shops 2 and 4)} \\
& \quad + 80x_{2,2}x_{4,1} + 130x_{2,2}x_{4,3} + 100x_{2,2}x_{4,4} \\
& \quad + 150x_{2,3}x_{4,1} + 130x_{2,3}x_{4,2} + 120x_{2,3}x_{4,4} \\
& \quad + 170x_{2,4}x_{4,1} + 100x_{2,4}x_{4,2} + 120x_{2,4}x_{4,3})
\end{aligned}$$

$$3(80x_{3,1}x_{4,2} + 150x_{3,1}x_{4,3} + 170x_{3,1}x_{4,4} \quad \text{(shops 3 and 4)}$$
$$+ 80x_{3,2}x_{4,1} + 130x_{3,2}x_{4,3} + 100x_{3,2}x_{4,4}$$
$$+ 150x_{3,3}x_{4,1} + 130x_{3,3}x_{4,2} + 120x_{3,3}x_{4,4}$$
$$+ 170x_{3,4}x_{4,1} + 100x_{3,4}x_{4,2} + 120x_{3,4}x_{4,3})$$

$$\begin{aligned}
\text{s.t.} \quad & x_{1,1} + x_{1,2} + x_{1,3} + x_{1,4} = 1 && \text{(1, Clothes Are)} \\
& x_{2,1} + x_{2,2} + x_{2,3} + x_{2,4} = 1 && \text{(2, Computers Aye)} \\
& x_{3,1} + x_{3,2} + x_{3,3} + x_{3,4} = 1 && \text{(3, Toy Parade)} \\
& x_{4,1} + x_{4,2} + x_{4,3} + x_{4,4} = 1 && \text{(4, Book Bazaar)} \\
& x_{1,1} + x_{2,1} + x_{3,1} + x_{4,1} = 1 && \text{(location 1)} \\
& x_{1,2} + x_{2,2} + x_{3,2} + x_{4,2} = 1 && \text{(location 2)} \\
& x_{1,3} + x_{2,3} + x_{3,3} + x_{4,3} = 1 && \text{(location 3)} \\
& x_{1,4} + x_{2,4} + x_{3,4} + x_{4,4} = 1 && \text{(location 4)} \\
& x_{i,j} = 0 \text{ or } 1 \quad i = 1, \ldots, 4; j = 1, \ldots, 4
\end{aligned}$$

ed until we
aracteristic

The objective function computes total flow distance for all pairs of shops and all possible assigned locations. Assignment constraints assure that one shop goes to each location and each locations gets one shop. An optimal assignment places shop 1 in location 1, shop 2 in location 4, shop 3 in location 3, and shop 4 in location 2, for a total flow distance of 3260 thousand customer-feet.

SAMPLE EXERCISE 11.11: FORMULATING QUADRATIC ASSIGNMENT MODELS

An industrial engineer has divided a proposed machine shop's floor area into 12 grid squares, g, each of which will be the location of a single machine m. He has also estimated the distance, $d_{g,g'}$, between all pairs of grid squares and the number of units, $f_{m,m'}$, that will have to travel between machines m and m' (in both directions) during each week of operation. Formulate a quadratic assignment model to layout the shop in a way that will minimize material handling cost (i.e., minimize the product of between machine flows and the distance between their locations). Assume $d_{g,g'} = d_{g',g}$.

***Modeling*:** Using the decision variables

(11.12)

$$x_{m,g} \triangleq \begin{cases} 1 & \text{if machine } m \text{ is located at grid square } g \\ 0 & \text{otherwise} \end{cases}$$

the required model is

$$\begin{aligned}
\min \quad & \sum_{m=1}^{12} \sum_{g=1}^{12} \sum_{m'>m}^{12} \sum_{\substack{g'=1 \\ g' \neq g}}^{12} f_{m,m'} d_{g,g'} x_{m,g} x_{m',g'} && \text{(flow distance)} \\
\text{s.t.} \quad & \sum_{g=1}^{12} x_{m,g} = 1 \quad m = 1, \ldots, 12 && \text{(square per machine)} \\
& \sum_{m=1}^{12} x_{m,g} = 1 \quad g = 1, \ldots, 12 && \text{(machine per square)} \\
& x_{i,j} = 0 \text{ or } 1 \quad m = 1, \ldots, 12; \quad g = 1, \ldots, 12
\end{aligned}$$

非线性規劃 LINGO

e.g. Max $Z = 2x_1^2 + 3x_2^2$

s.t.

$x_1 + x_2 \leq 2$

$x_1, x_2 \geq 0$

```
MODEL:
MAX = 2.0 * X1 ^ 2 + 3.0 * X2 ^ 2;
    X1 + X2 <= 2 ;
END
```

如有其他变数選項加在 END之前

@BIN(X1) 令变数 X1 為 {0, 1}

@GIN(X1) 令变数 X1 為 整数

@FREE(X1) 解除 default $X1 \geq 0$

指派問題

$$\text{Min } Z=\sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij} X_{ij} \quad \left(\text{Max } Z=\sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij} X_{ij}\right)$$

成本　　　　　　　　　　效率

$$\text{s.t. } \sum_{i=1}^{n} X_{ij}=1 \quad (j=1,\ldots,n)$$

$$\sum_{j=1}^{n} X_{ij}=1 \quad (i=1,\ldots,n)$$

可以 K. W. Kuhn 提出的匈牙利法 (Hungarian Method) 求解。

	趙	錢	孫	李	王
仰式	37	32	(33)	37	35
蝶式	43	33	42	(34)	41
蛙式	33	(28)	38	30	33
自由式	(29)	26	29	28	31

Total Cost = 124

```
min 37x11+32x12+33x13+37x14+35x15+
    43x21+33x22+42x23+34x24+41x25+
    33x31+28x32+38x33+30x34+33x35+
    29x41+26x42+29x43+28x44+31x45
st
   x11+ x12+ x13+ x14+ x15=1
   x21 +x22+ x23+ x24+ x25=1
   x31 +x32+ x33+ x34+ x35=1
   x41 +x42+ x43+ x44+ x45=1
   x11+ x21+ x31+ x41 <=1
   x12+ x22+ x32+ x42 <=1
   x13+ x23+ x33+ x43 <=1
   x14+ x24+ x34+ x44 <=1
   x15+ x25+ x35+ x45 <=1
end
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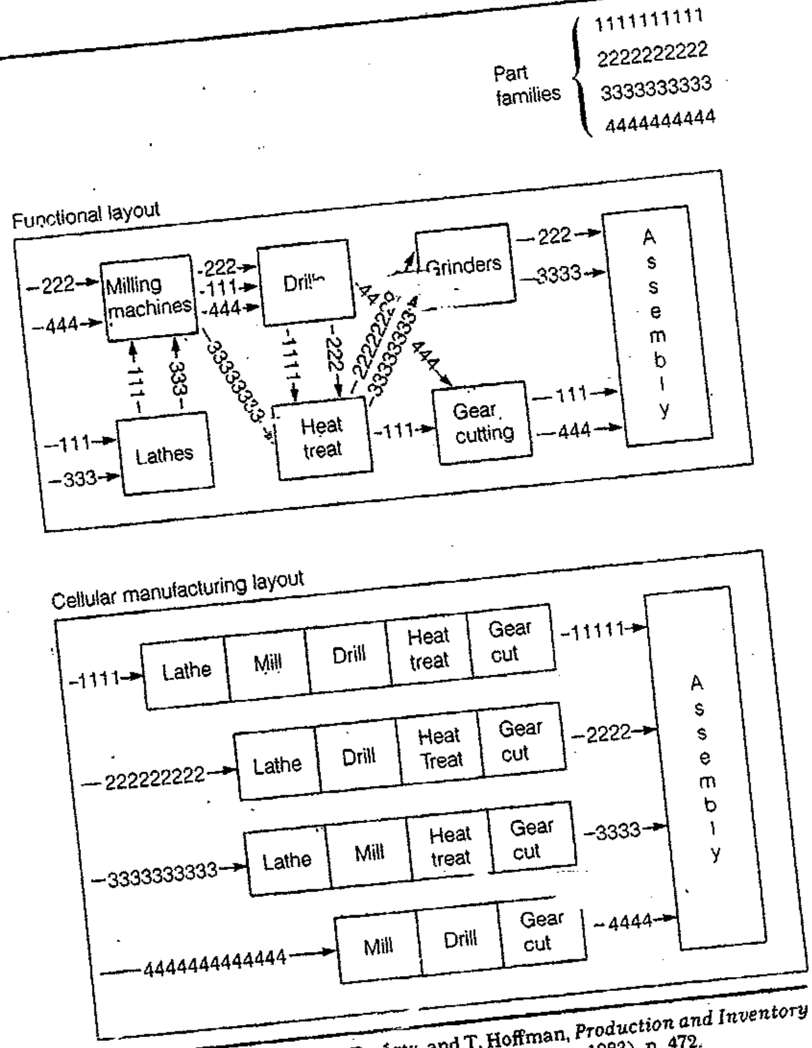
FIGURE 7–8

Comparison of Functional and Cellular Manufacturing Layouts



Source: Adapted from D. Fogarty, and T. Hoffman, *Production and Inventory Management* (Cincinnati: South-Western Publishing, 1983), p. 472.

relate to the grouping of equipment. They include faster throughput time, less material handling, less work in process inventory, and reduced setup time.

Group Technology

In order to use cellular manufacturing, there must be groups of items that have similar processing characteristics. Moreover, these items must be identified. The grouping process is known as *group technology,* which involves indentifying items that have similarities in either *design characteristics* or in *manufacturing character-*