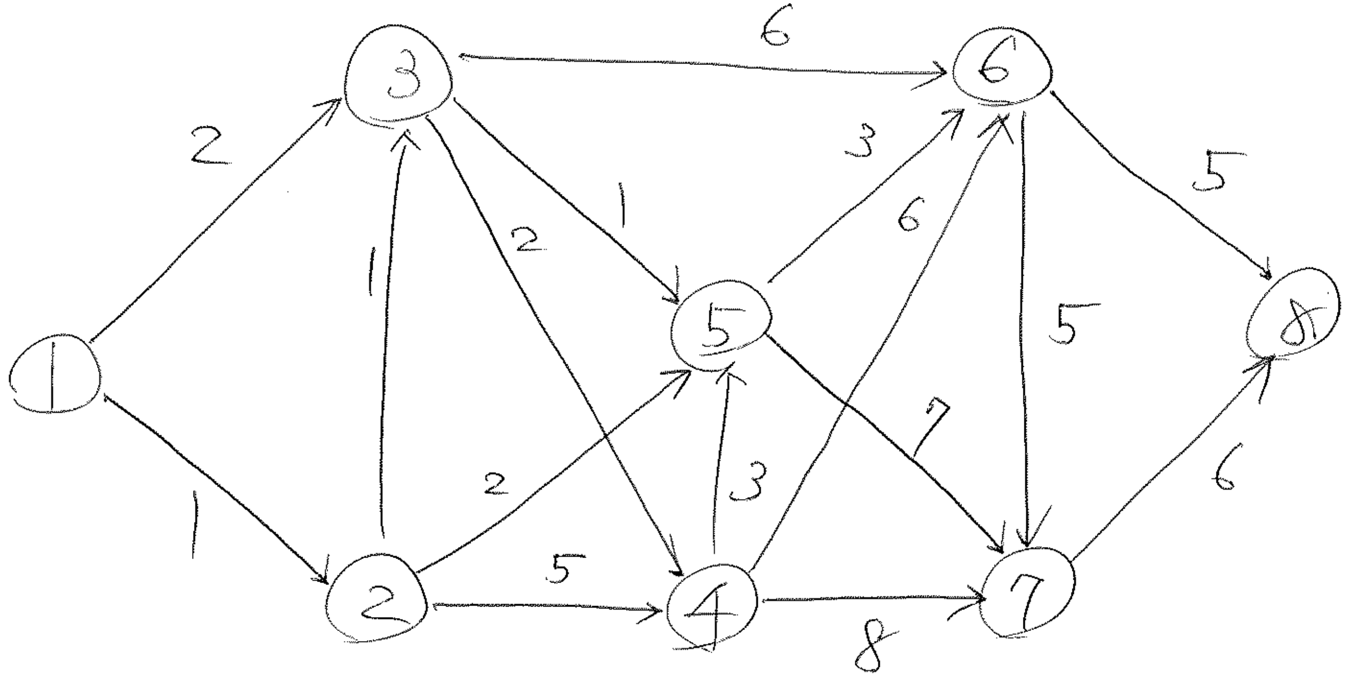
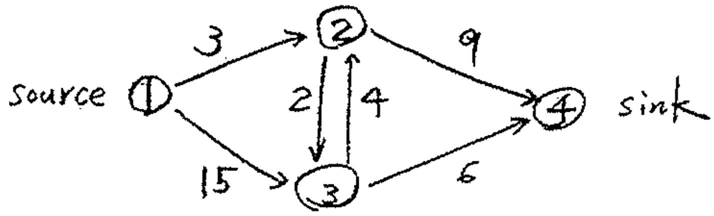

1
2
3
4
5
6
7
8
2
1
1
6
1
2
2
5
3
6
3
7
8
5
5
6

FORMULATING SHORTEST PATHS AS NETWORK FLOWS



$$\min \; 3x_{1,2} + 15x_{1,3} + 2x_{2,3} + 9x_{2,4} + 4x_{3,2} + 6x_{3,4}$$

s.t.

$$-x_{1,2} - x_{1,3} = -1 \quad \text{(NODE 1)}$$

$$x_{1,2} + x_{3,2} - x_{2,3} - x_{2,4} = 0 \quad \text{(NODE 2)}$$

$$x_{1,3} + x_{2,3} - x_{3,2} - x_{3,4} = 0 \quad \text{(NODE 3)}$$

$$x_{2,4} + x_{3,4} = 1 \quad \text{(NODE 4)}$$

$$x_{1,2},\ x_{1,3},\ x_{2,3},\ x_{2,4},\ x_{3,2},\ x_{3,4} \in \{0, 1\}.$$

$$x_{i,j} = \begin{cases} 1 & \text{arc } i,j \text{ will be on optimal path} \\ 0 & \text{otherwise.} \end{cases}$$

Note: the coefficient of inflow arc +1
the coefficient of outflow arc −1
a supply of 1 at the source (−1)
and a demand of 1 at the sink (+1)

An optimal solution to the model is $x_{1,2} = x_{2,3} = x_{3,4} = 1$

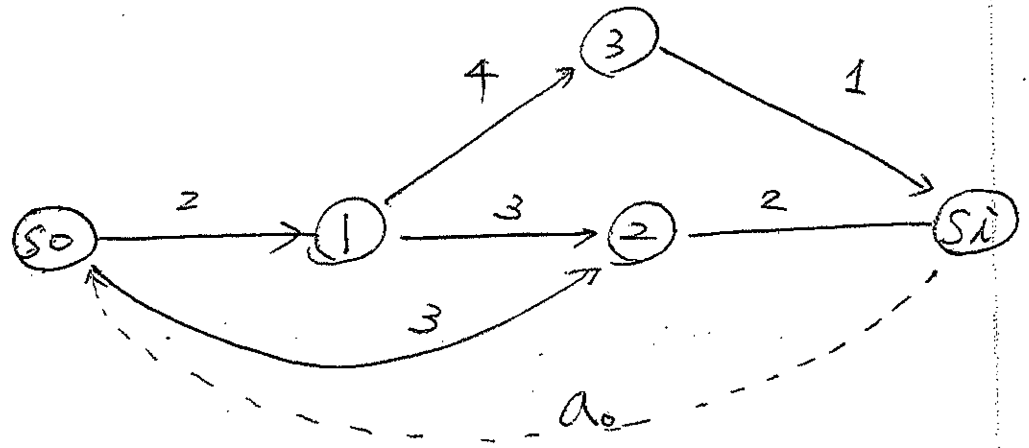
$x_{3,2} = x_{1,3} = x_{2,4} = 0$, The shortest path is

1-2-3-4

Maximum Flow Problems 最大流量(川流)問題.

在許多情況下，網路之節段可視為有容量限制去建構模式。在這些情形下，通常想從一 starting point 起点 (source 源点) 運送最大之流量至一 terminal point 終点 (sink 匯点). 這類之問題，諸如，自來水同一時間之最大流量，高速公路之路網最大流量等。

e.g.



Sunco Oil 公司想透过 pipeline 從 so 至 si 運送最大量的石油 (在每小時) 網路上之數字代表每小時可通过該節段之最大流量 (每小時百万 barrels.

由於管道的口徑大小不同. 最大流量問題

就是要決定，每小時從 S_0 至 S_i 最大流量。

令 X_{ij} 代表每小時會經过弧 (i, j) 百万 barrels.

X_0 : flow 經过 artificial arc, i.e. 進入 sink 的流量 (a_0)

$$\text{Max} \quad Z = X_0$$

s.t.

$$X_{s_0,1} \le 2$$
$$X_{s_0,2} \le 3$$
$$X_{1,2} \le 3$$
$$X_{2,s_i} \le 2$$
$$X_{1,3} \le 4$$
$$X_{3,s_i} \le 1$$

(Arc capacity constraints)

---

conservation-of-flow constraint

$$X_0 = X_{s_0,1} + X_{s_0,2} \quad \text{Node } s_0 \text{ flow constraint}$$
$$X_{s_0,1} = X_{12} + X_{13} \quad \text{Node 1 flow constraint}$$
$$X_{s_0,2} + X_{12} = X_{2,s_i} \quad \text{Node 2 flow ''}$$
$$X_{13} = X_{3,s_i} \quad \text{Node 3 flow ''}$$
$$X_{3,s_i} + X_{2,s_i} = X_0 \quad \text{Node } s_i \text{ flow ''}$$

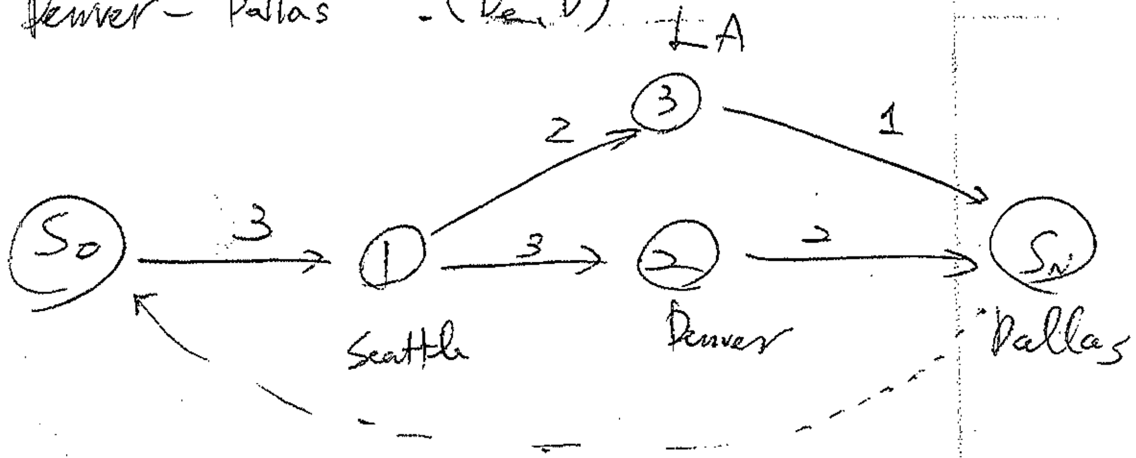
$$X_{ij} \ge 0$$

Optimal $Z=3$, $X_{s_0,1} = 2$, $X_{s_0,2} = 1$, $X_0 = 3$

$X_{1,3} = 1$ $\quad X_{3,s_i} = 1$

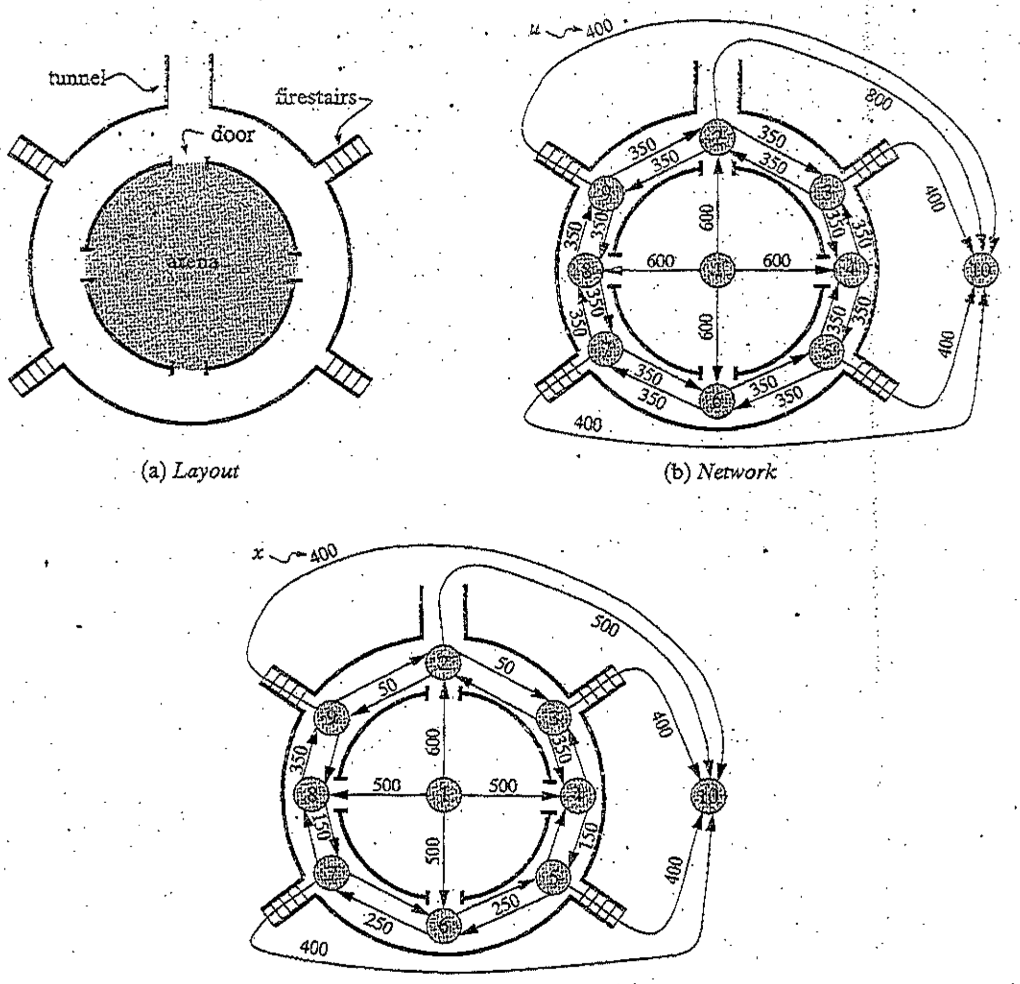
e.g.

Cities	Maximum Number of Daily Flights
Juneau – Seattle (J, S)	3
Seattle – L.A. (S, L)	2
Seattle – Denver (S, De)	3
LA – Dallas (L, D)	1
Denver – Dallas (De, D)	2



$Z = X_0 = 3 \quad X_{JS} = 3 \quad X_{SL} = 1 \quad X_{SDE} = 2 \quad X_{LD} = 1 \quad X_{DE,D} =$

Fly-by-Night Airlines 必需決定 has many connecting flights can be arranged between Juneau, Alaska and Dallas, Texas, 由於 Landing space 有限, 每日可降落之最大班次如表顯示。Setup a maximum flow problem to maximize the number of connecting flights daily from Juneau to Dallas.



(a) *Layout* (b) *Network*

(c) *Optimal flow*

FIGURE 10.14 Building Evacuation Maximum Flow Example

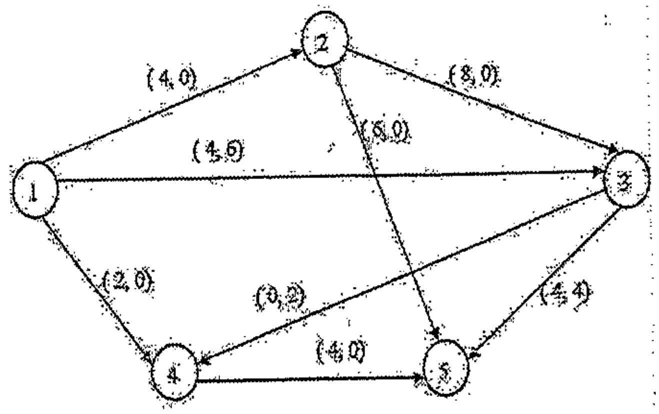
Return Arc Network Flow Formulation of Maximum Flow Problems

As so far presented, neither the tiny example of Sample Exercise 10.17 nor the larger one of Figure 10.14(b) is a minimum cost network flow problem. Flow conservation and capacity requirements are much like standard model 10.3, but we have specified no costs, and flows do not balance at source and sink.

To create a true minimum cost flow problem, we add a return arc.

10.51 **Return arcs** balance unknown source-to-sink flows by feeding back that flow in an artificial arc from sink to source.

Adding a return arc to the maximum flow example of Figure 10.14(b) produces the following digraph:



最大流量問題可用下述演算法求解：

步驟1　找出一條由源點至滙點可容納一正流量物品的路徑。若不存在，跳至步驟5。

步驟2　算出沿這條路徑可裝運的最大容量，並以 k 表示之。

步驟3　這條路徑中，每一分枝的順向容量（即順著 k 單位流量的方向之容量）均減少 k 單位，而逆向容量均增加 k。滙點的運送量也增加 k 單位。

步驟4　回到步驟1。

步驟5　最大流量即爲滙點的運送量。比較原網路與終期網路，將任一容量的減少視爲裝運，則可決定最優裝運計畫。

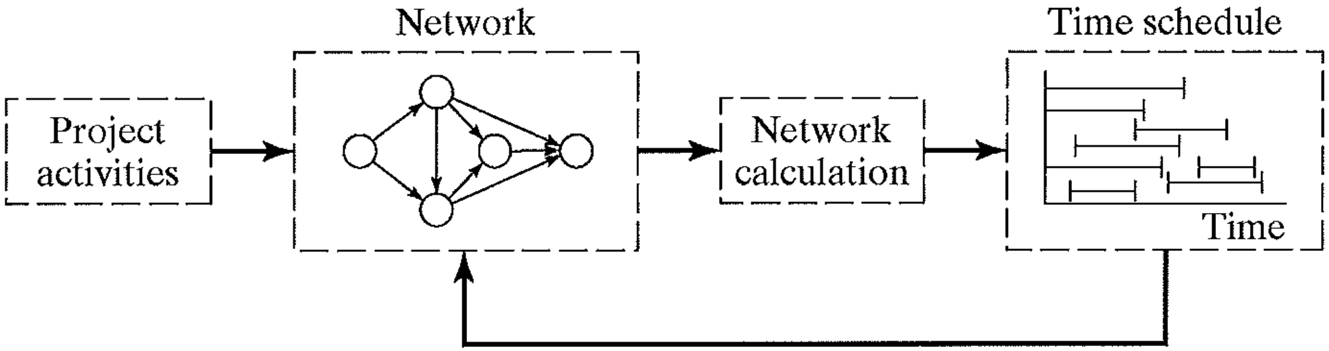


Figure 6.38

Phases for project planning with CPM-PERT.

Three rules are available for constructing the network.

Rule 1. Each activity is represented by one, and only one arc.

Rule 2. Each activity must be identified by two distinct end nodes.

Rule 3. To maintain the correct precedence relationships, the following questions must be answered as each activity is added to the network:

(a) What activities must immediately precede the current activity?

(b) What activities must follow the current activity?

(c) What activities must occur concurrently with the current activity?

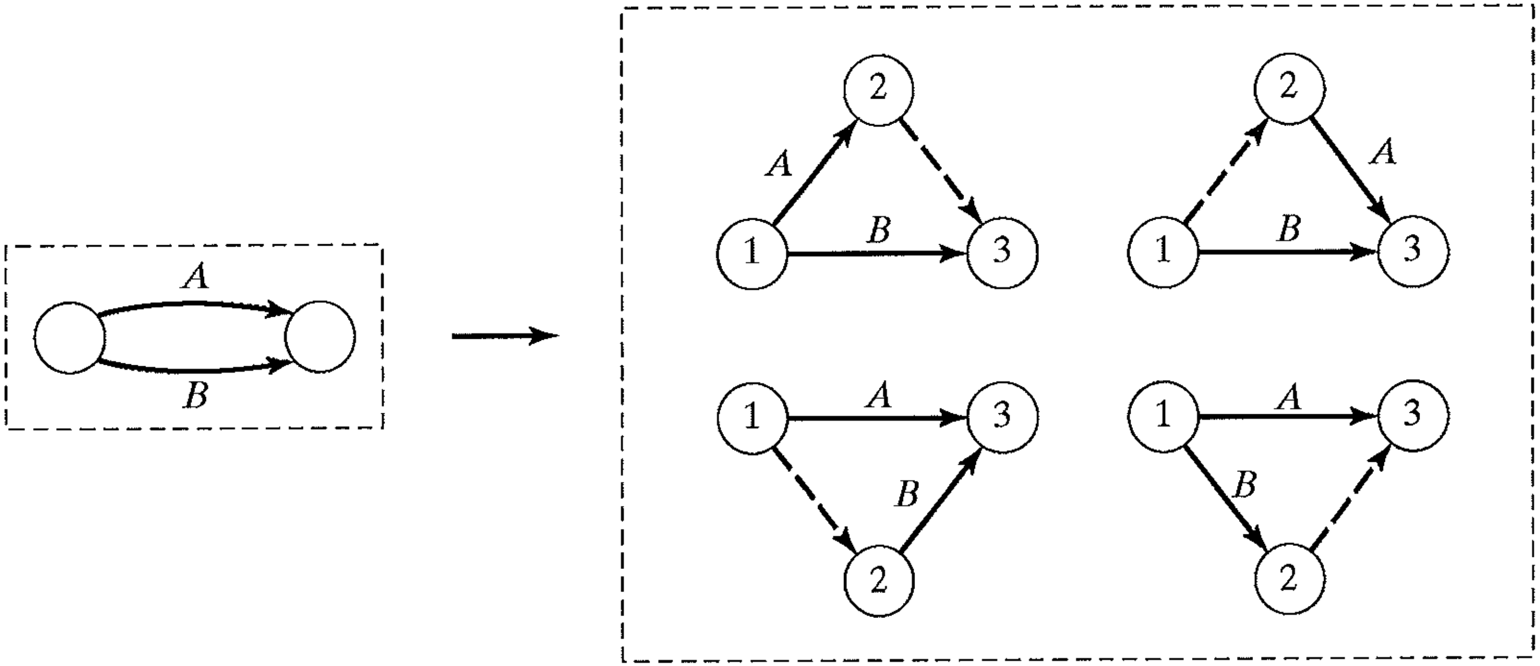
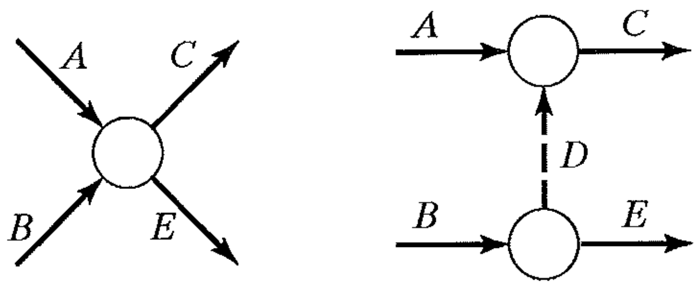


Figure 6.39

Use of dummy activity to produce unique representation of concurrent activities.

consumes no time and resources



(a) (b)

Figure 6.40

Use of dummy activity to ensure correct precedence relationship.

Example 6.5-1

一出版商連繫作者出版教科書，其相関活動如下：

Activity	Predecessor(s)	(weeks) Duration
A : Manuscript proofreading by editor	—	3
B : Sample pages preparation	—	2
C : Book cover design	—	4
D : Artwork preparation	—	3
E : Arthor's approval of edited manuscript and sample pages	A, B	2
F : Book formatting	E	4
G : Arthor's review of formatted pages	F	2
H : Arthor's review of artwork	D	1
I : Production of printing plates	G, H	2
J : Book production and binding	C, I	4

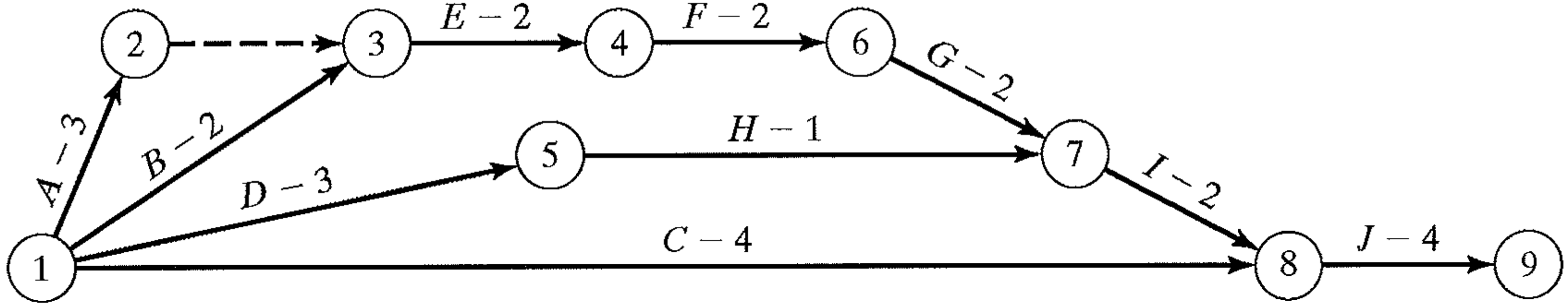


Figure 6.41

Project network for Example 6.5-1.

要徑的計算

✓ **Forward Pass**

計算各活動之最早開始時間(The Earliest Start Time-ES) 將其置放□中。

最早開始時間的決定是所有活動的最早完成時間(The Earliest Finish Time-EF)選取最大。

✓ **Backward Pass**

計算各活動之最晚完成時間(The Latest Finish Time-LF) 將其置放Δ中。

最晚完成時間的決定是所有活動的最晚開始時間(The Latest Start Time-LS)選取最小。

✓滿足要徑的條件

$ES_i=LF_i$

$ES_j=LF_j$

$ES_j-ES_i=LF_j-LF_i=D_{ij}$

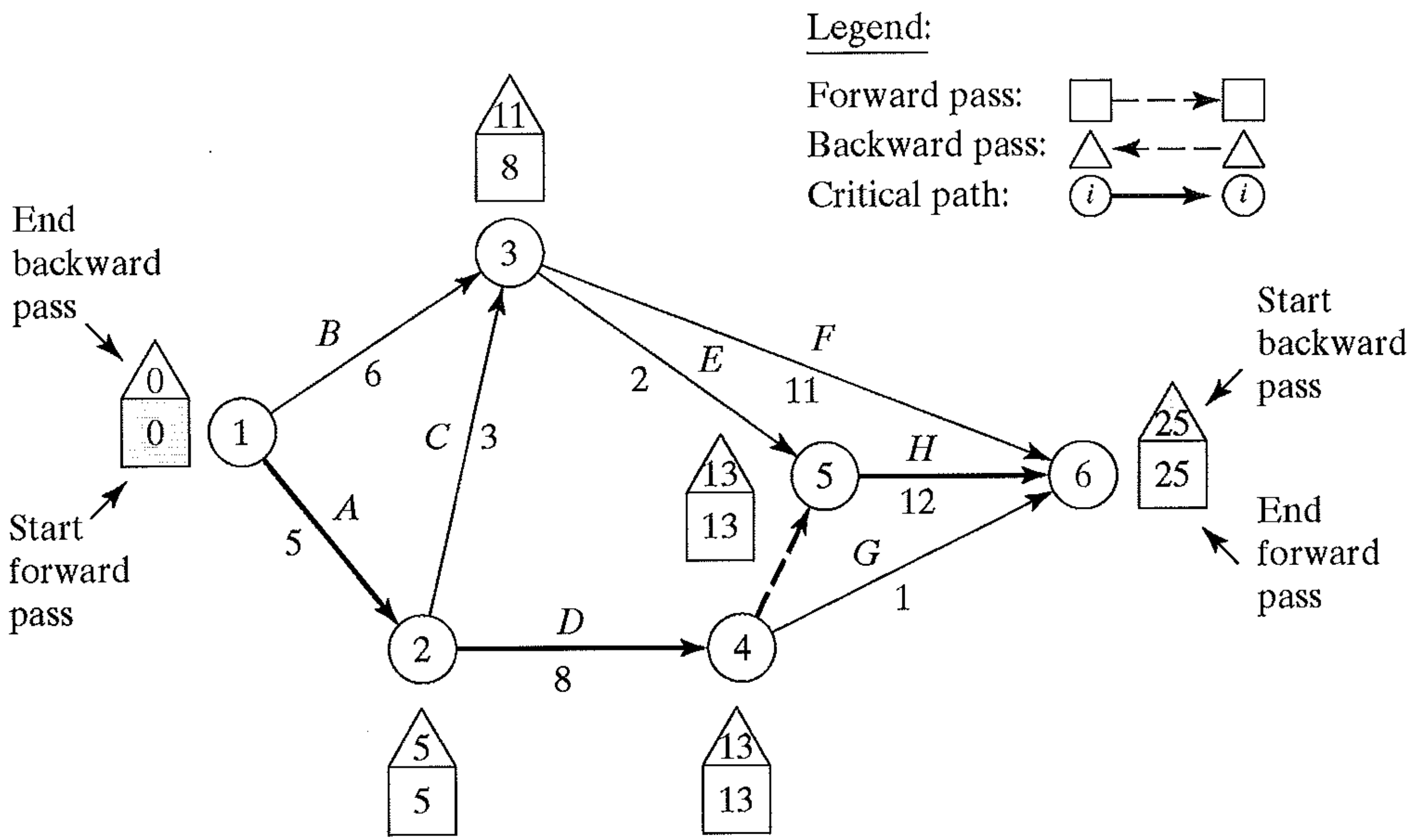
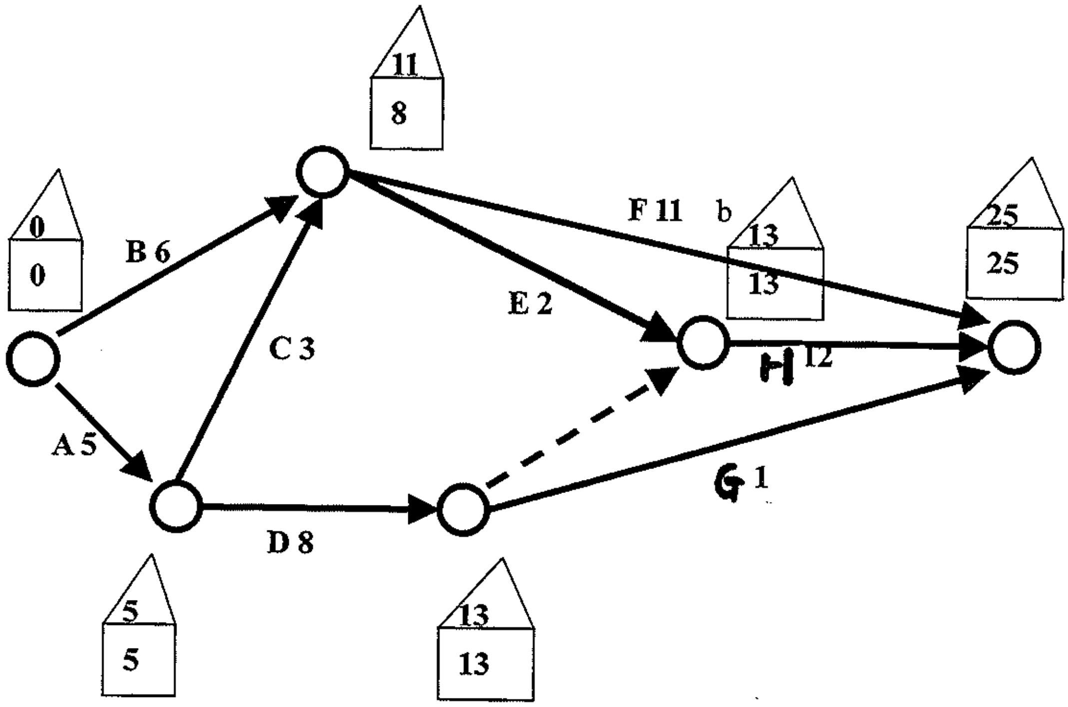


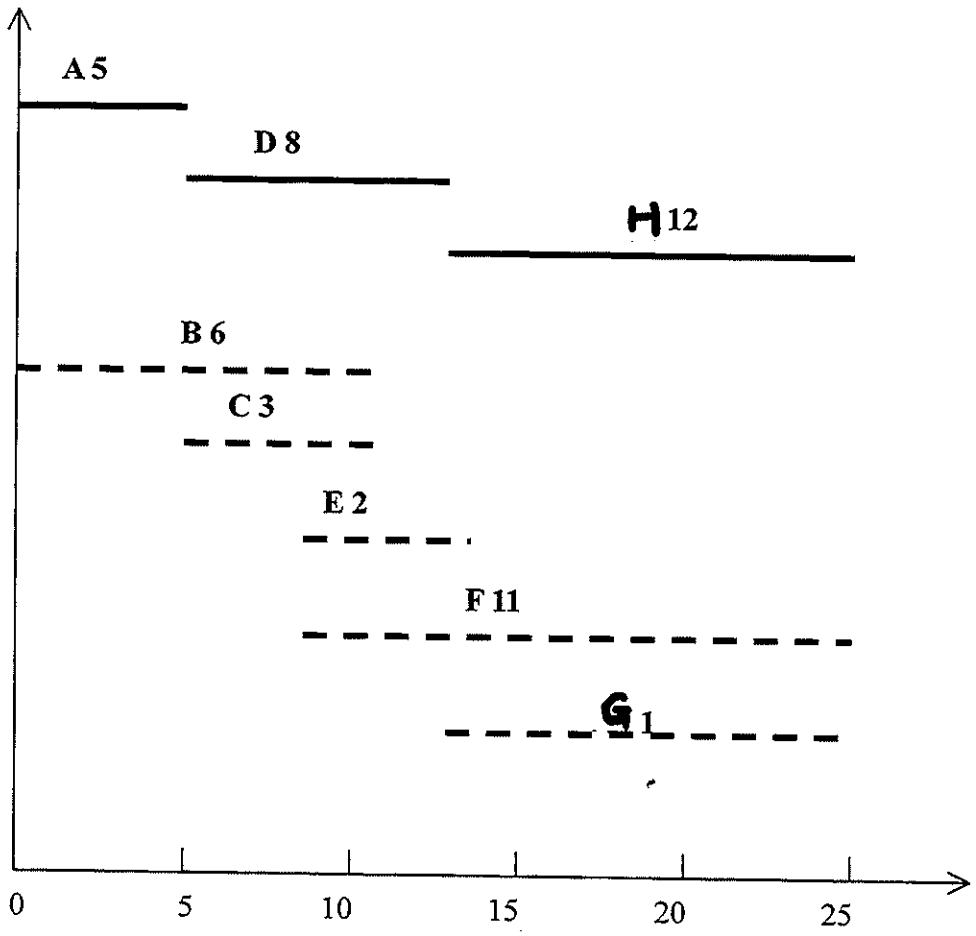
Figure 6.42

Forward and backward pass calculations for the project of Example 6.5-2.

構建時間表



11
8
0
0
B 6
F 11
b
13
13
25
25
E 2
C 3
H 12
A 5
G 1
D 8
5
5
13
13



A 5
D 8
H 12
B 6
C 3
E 2
F 11
G 1
0
5
10
15
20
25

活動的控制與浮時的計算

非要徑活動	TF	FF
B	5	2
C	3	0
E	3	3
F	6	6
G	11	11

B 活動所能延後之時間，不能超過(**FF=2**)兩小時。

- **Total Float=LF_j-ES_i-D_{ij}**

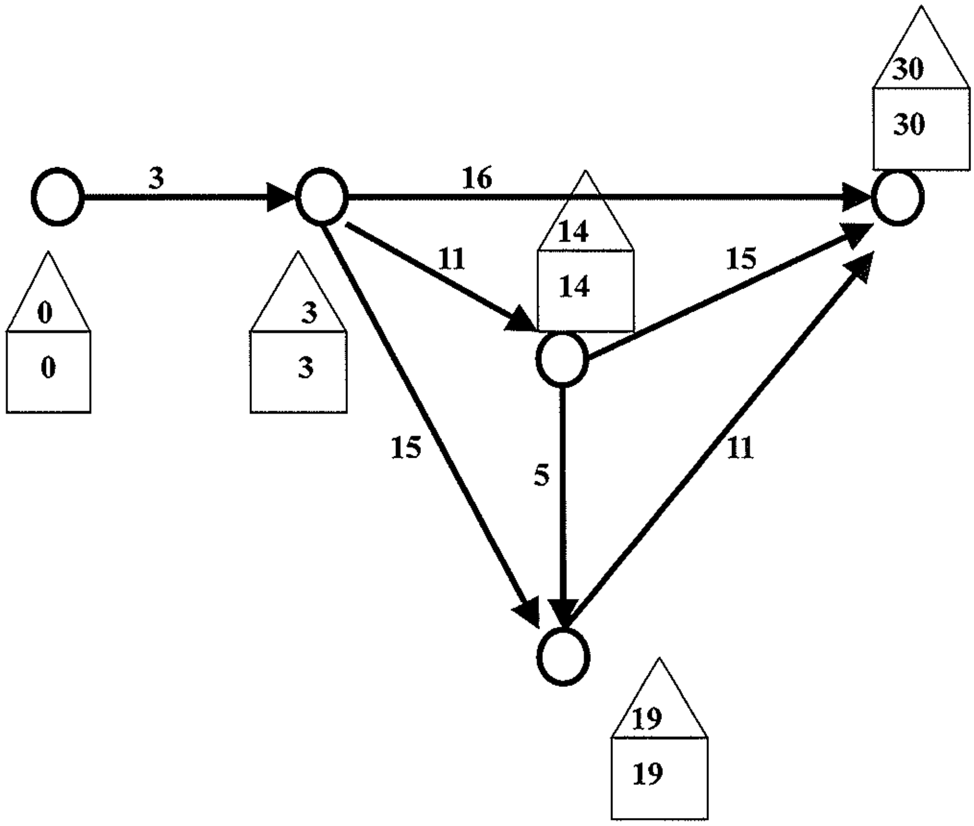
  總浮時:最大可利用時間與執行時間之間的差

- **Free Float=ES_j-ES_i-D_{ij}**

  自由浮時:所有活動儘早開始與執行時間之間的差

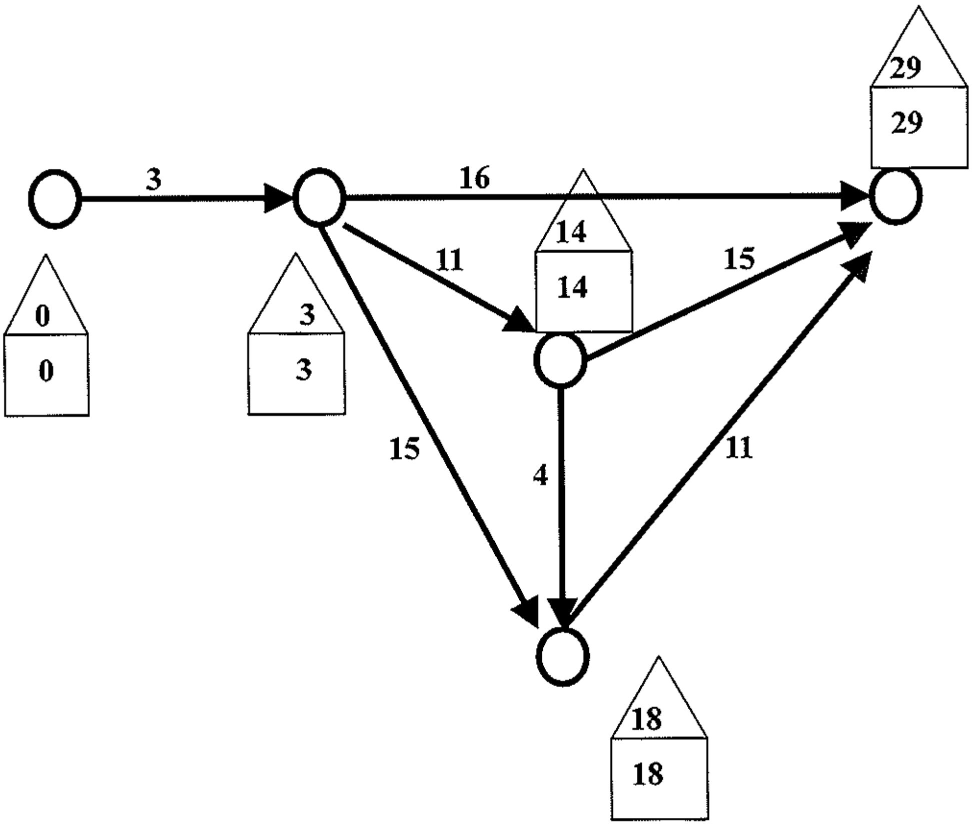
趕工

工作	正常時間	趕工時間	正常成本	趕工成本	斜率
1-2	3	2	500	650	150
2-3	11	8	220	280	20
2-4	15	8	500	850	50
2-5	16	15	275	300	25
3-4	5	3	30	60	15
3-5	15	12	600	720	40
4-5	11	9	400	440	20



完工日數 30 天，成本 2525，欲趕工一日，則選擇活動(3,4)，因其成本為 15 最低。

趕工(續)



如今要徑有三條：

1-2-3-5　　1-2-3-4-5　　1-2-4-5

欲趕工一日，必須在三條要徑上，同時減少一日，考慮不同的成本組合如下：

(1, 2)	**150** 元
(2, 3) (2, 4)	**20+50=70** 元
(2, 4) (3, 4) (3, 5)	**50+15+40=105** 元
(3, 5) (4, 5)	**40+20=60** 元　✓

PERT(Program Evaluation Review Technique 計劃評核術)

a 樂觀時間

m 最可能時間

b 悲觀時間

$$te = \frac{a + 4m + b}{6}$$ 活動期望時間

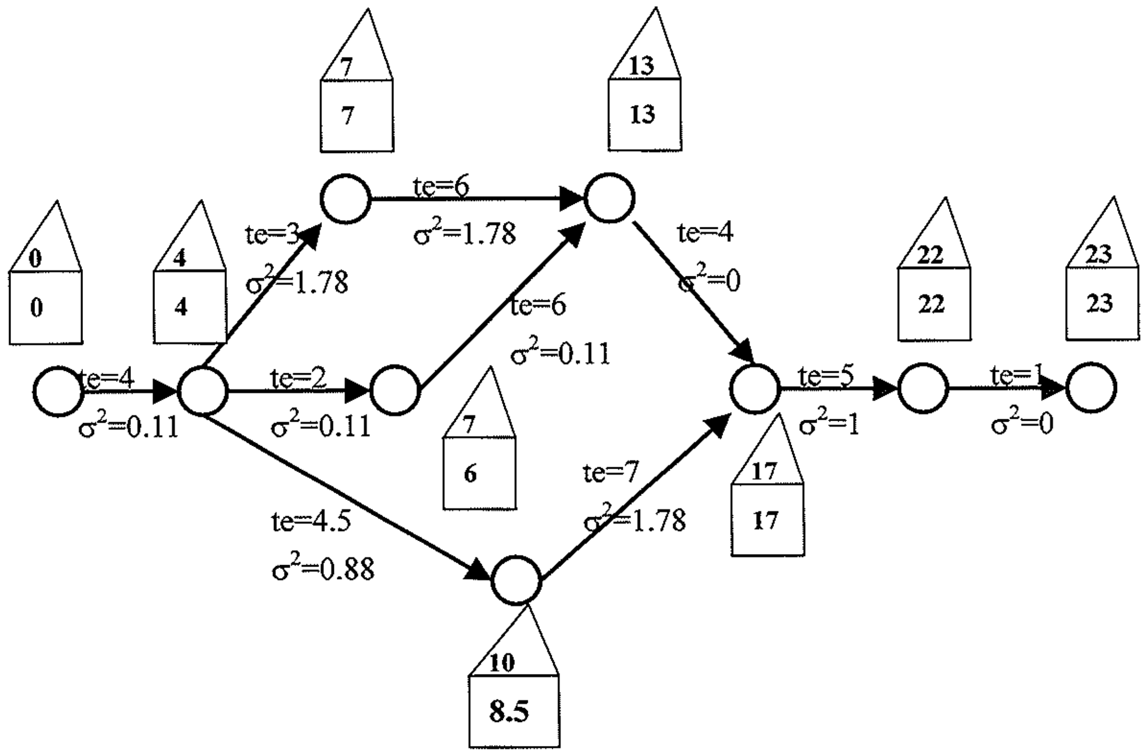
$$\sigma^2 = \frac{(b-a)^2}{36}$$ 活動時間的變異數

假設：

1. 各個活動時間是統計獨立
2. 在要徑上的總期望時間及總時間變異數，為各活動的期望時間與變異數之和
3. 由中央極限定理，假設完工時間是常態分配

PERT 範例

代號	先行作業	樂觀	最可能	悲觀	t_e	σ^2
A	-	3	4	5	4	0.11
B	A	1	2	3	2	1.78
C	A	3	4	8	4.5	0.11
D	A	1	2	9	3	0.69
E	D	4	5	12	6	1.78
F	B	5	6	7	8	0.11
G	E F	4	4	4	4	0
H	C	5	6	13	7	1.78
I	G H	4	4	10	5	1
J	I	1	1	1	1	0



PERT 範例(續)

由前例算出要 23 個月完成，如果限定專案 22 個月需完成，則其完成機率為何？

$$z = \frac{\text{限定時間} - \text{期望要徑時間}}{\text{沿要徑標準差}}$$

$$= \frac{22 - 23}{\sqrt{0.11 + 1.78 + 1.78 + 0 + 1 + 0}} = -0.463$$

查表其機率約為 0.3228

一般能否如期完工，其機率 P

0.25 ≥ **P** ≥ 0 在既定資源內不可能如期完工

0.6 ≥ **P** > 0.25 限定時間是可以接受的，所以專案可開始

1 ≥ **P** > 0.6 最佳情況，可以將一部分資源轉為其他用途