

$$x_1 + x_2 \leq 5$$

either-or
constraint

$$x_1 + 2x_2 \leq 7$$

(恰有一成)
互

$$\begin{aligned} x_1 + x_2 &\leq 5 + My \\ x_1 + 2x_2 &\leq 7 + M(1-y) \\ y &\in \{0, 1\} \end{aligned}$$

$$\begin{aligned} x_1 + x_2 &\leq 5 + My \\ x_2 + 2x_2 &\leq 7 + My \\ y_1 + y_2 &= 1 \\ y_1, y_2 &\in \{0, 1\} \end{aligned}$$

$$x_1 + x_2 \leq 5$$

either-or

$$3x_1 + 2x_2 \geq 10$$

$$\begin{aligned} x_1 + x_2 &\leq 5 + My \\ -3x_1 - 2x_2 &\leq -10 + M(1-y) \\ y &\in \{0, 1\} \end{aligned}$$



$$\begin{aligned} x_1 + x_2 &\leq 5 + My \\ 3x_1 + 2x_2 &\geq 10 - My \\ y_1 + y_2 &= 1 \\ y_1, y_2 &\in \{0, 1\} \end{aligned}$$

at most one equation is satisfied

至多一式成立

$$x_1 + x_2 \leq 5$$

$$x_1 + 2x_2 \leq 7$$

$$\begin{aligned} x_1 + x_2 &\leq 5 + My \\ x_1 + 2x_2 &\leq 7 + My \\ y_1 + y_2 &\geq 1 \\ y_1, y_2 &\in \{0, 1\} \end{aligned}$$

$$x_1 + x_2 = 5$$

at least one equation is satisfied

$$x_1 + 2x_2 = 7$$

$$x_1 + x_2 \leq 5$$

$$x_1 + 2x_2 \leq 7$$

$$10x_1 - x_2 \geq 1$$

at least two equations are satisfied.

$$x_1 + x_2 \leq 5 + M/y_1$$

$$x_1 + 2x_2 \leq 7 + M/y_2$$

$$x_1 + y_2 \leq 1$$

$$y_1, y_2 \in \{0, 1\}$$

$$x_1 + x_2 \leq 5 + M/y_1$$

$$x_1 + 2x_2 \leq 7 + M/y_2$$

$$10x_1 - x_2 \geq 1 - M/y_3$$

$$x_1 + y_2 + y_3 \leq 1$$

$$y_1, y_2, y_3 \in \{0, 1\}$$

$x_1 = 2$ or 4 or 6 One of them is satisfied

$$x_1 = 2y_1 + 4y_2 + 6y_3$$

$$y_1 + y_2 + y_3 = 1$$

$$y_1, y_2, y_3 \in \{0, 1\}$$

$|2x_1 + x_2 + x_3| = 2$ or 4 or 6 One of them is satisfied

$$\Rightarrow x_1 + x_2 + x_3 = -2y_1 - 4y_2 - 6y_3 + 2y_4 + 4y_5 + 6y_6$$

$$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 = 1$$

$$y_1, y_2, y_3, y_4, y_5, y_6 \in \{0, 1\}$$

All-or-nothing

$$0 \leq x_1 \leq 30 \Rightarrow \overset{\text{希望}}{x_1=0} \text{ or } x_1=30$$
$$x_1 = 30 y_1 \quad y_1 \in \{1, 0\}$$

$$x_1 = 0 \text{ or } x_1 \geq 30$$

$$x_1 \geq 30 y_1$$

$$x_1 \leq M y_1 \quad y_1 \in \{1, 0\}$$

If-then constraint

$$\text{If } f(x_1, x_2, \dots, x_n) > 0 \text{ then}$$
$$g(x_1, x_2, \dots, x_n) \geq 0$$

$$-g(x_1, x_2, \dots, x_n) \leq M y$$

$$f(x_1, x_2, \dots, x_n) \leq M(1-y)$$

Linearizing Minimax and Maximin Objective function

$$\text{Min } Z = \max \{x_1 + x_2, -x_1 + 2x_2\}$$

$$\text{st. } x_1 + x_2 \leq 5$$

$$x_1, x_2 \geq 0$$

$$\text{Min } Z = f$$

$$f \geq x_1 + x_2$$

$$f \geq -x_1 + 2x_2$$

$$x_1 + x_2 \leq 5$$

$$x_1, x_2 \geq 0$$

$$\text{Max } Z = \min \{x_1 + x_2, -x_1 + 2x_2\}$$

$$\text{st. } x_1 + x_2 \leq 5$$

$$x_1, x_2 \geq 0$$

$$\text{Max } Z = f$$

$$f \leq x_1 + x_2$$

$$f \leq -x_1 + 2x_2$$

$$x_1 + x_2 \leq 5$$

$$x_1, x_2 \geq 0$$

Min deviation objective function

$$\text{Min } 4|x_1 - x_2| + 4|y_1 - y_2|$$

Let S_i^+ = positive difference in absolute value term i
 S_i^- = negative difference in absolute value term i

$$\begin{array}{l} \text{Min } 4(S_1^+ + S_1^-) + 4(S_2^+ + S_2^-) \\ \text{st.} \\ x_1 - x_2 = S_1^+ - S_1^- \\ y_1 - y_2 = S_2^+ - S_2^- \end{array}$$

$$\text{Max } Z = |x_1 + x_2 + x_3|$$

$$\text{st. } 2x_1 + 3x_2 - x_3 \leq 10$$

$$\begin{array}{l} \text{Max } Z = S_1^+ + S_1^- \\ \text{st. } x_1 + x_2 + x_3 = S_1^+ - S_1^- \\ 2x_1 + 3x_2 - x_3 \leq 10 \\ x_1, x_2, x_3, S_1^+, S_1^- \geq 0 \end{array}$$